RUM, a recursive update putty-semi-putty vintage production model: sectoral estimation results for Germany and the Netherlands

*RUM , a **R**ecursive **U**pdate Putty-Semi-Putty Vintage Production **M**odel :*

Sectoral Estimation Results For Germany and the Netherlands

by

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1 Introduction

In this paper we present the estimation results obtained using the RUM putty-semi-putty vintage production model as described in more detail in van Zon (1994). 'RUM' stands for 'Recursive Update Model', a name which is rather general, but which refers to the fact that one of the central features of the model is that its behaviour at the aggregate level is obtained by means of a set of straight forward recursive update rules which are applied at a more disaggregated level. The application of these rules in the context of a direct-search estimation procedure has enormous practical advantages over the specification of a full vintage model. We will come back to this later.

The principle reasons why we developed the RUM variant of a putty-semi-putty vintage production model instead of using an aggregate production function approach are **first** that we feel that technological change does not fall as manna from heaven : it must be bought and paid for. **Secondly**, many of the improvements in production processes are linked with embodied technological change, while **third** 'standard' vintage model estimation procedures are relatively tedious and computer time-consuming and are a practical barrier to a successful application of vintage models as part of a larger macro-sectoral model. We have tried to find a way to jump this barrier by means of taking an approximating shortcut, while retaining the idea of the embodiment of technological change and the 'technology induced' scrapping of old equipment. More in particular, we have tried to find a way to by-pass the cumbersome calculations involved in maintaining a complete 'book-keeping' account of all individual vintages which have come into existence 'almost from the beginning of time'.[1]

The RUM model is based on a putty-semi-putty vintage production structure. It is the existence of smooth substitution possibilities ex-post which enables us to compress the book-keeping account mentioned above in just a few equations which are directly relevant for aggregate behaviour. Putty-Clay and Clay-Clay vintage production models as they were first introduced by Johansen (1959) and Salter (1960) require such a book-keeping account when economic scrapping is endogenised. However, if the latter is not the case, then a more simple approach can be taken, as, for instance, the HERMES modellers have done (d'Alcantara and Italianer (1982)). The latter have assumed that scrapping is either only of the technical kind or economic scrapping takes place at a fixed rate next to technical decay. As Meijers and van Zon (1992) have shown, the performance of the HERMES production block can be improved upon by allowing, in principle at least, for endogenous scrapping. Hence, we prefer to use a specification which allows for variation in technical coefficients ex-post, and which includes the

1 The latter approach is 'standard practice' in the Dutch empirical vintage modelling tradition. See Den Hartog and Tjan (1976), Kuipers and van Zon (1982), Gelauff, Wennekers and de Jong (1985), and Muysken and van Zon (1987), for instance. A notable exception is Eigenraam (1987).

HERMES approach as a special case. The latter calls for the use of CES functions both ex-ante and ex-post since these contain the Leontief production function as a limiting case.

Contrary to the CRAPP-model (van Zon (1993)) which is a forerunner of RUM, the RUM model takes account of expectations regarding future prices and disembodied technological change in order to determine the best course of action needed to be taken today, both conditional on what has happened in the past and 'in response' to what is expected to happen in the future. Thus, present decisions build on decisions taken in the past, and are in line with the anticipations of future decisions. In short, we derive the principal features of the RUM model in a (partially) intertemporal setting, whereas the features of the CRAPP model were derived for the case of myopic behaviour.

Why then is the transition from myopic behaviour to non-myopic behaviour important? The reason is that we assume that substitution possibilities between labour and capital before the moment of installation of a new piece of equipment are larger than the substitution possibilities after the moment of installation. This implies that the choice of an initial technique uniquely defines the entire ex-post unit iso-quant. But since substitution possibilities ex-post are described by the ex-post unit-isoquant, this also implies that the ability to react to future (expected) changes in wage rates is also directly affected by the technical characteristics of the initial factor-mix. More in particular, the choice of a high labour/capital ratio in response to present wage conditions, for instance, diminishes one's opportunities to avoid future rises in wage costs which are associated with rises in wage-rates.

An important practical advantage of using the RUM model rather than a full putty-semi-putty vintage model lies in the relative ease by which it can be handled. RUM makes positive use of the fact that it is often not necessary to know all the details of every individual vintage : from a macro-economic point of view, only the average characteristics of the vintage capital stocks are important.

RUM uses a set of recursive update rules which describe the evolution over time of aggregate capital productivity and the aggregate capital/labour ratio in function of the ex-post substitution characteristics of the 'old' machinery and of the new machinery just installed. More in particular, we define a set of update rules which describe changes in the average characteristics of the vintage capital stock both in terms of the changes in the technological characteristics of new equipment (due to factor substitution ex-ante and embodied technological change) and in terms of the changes in the characteristics of the 'old' equipment (due to factor substitution ex-post and due to disembodied technological change). Thus, RUM avoids the tedious vintage book-keeping exercises mentioned earlier. Nonetheless, RUM is still able to imitate a full putty-semi-putty vintage model in an almost perfect way.

The set-up of this paper is as follows. In section 2 we will describe the features of the RUM model. In section 3 we will briefly sketch the estimation procedures which we have used in order to estimate the parameters of RUM for different sectors of industry for Germany and the Netherlands, as well as the sectoral classification we have employed. Moreover, we describe the data-sources we have used and the preliminary operations we have performed upon these data. Section 4 provides an overview of the estimation results for Germany and the Netherlands. Section 5 contains a summary and some concluding remarks.

2 The RUM Model

2.1 General Assumptions

We assume that there are two factors of production, labour and capital, which can be substituted both ex-ante and ex-post. We also assume that substitution possibilities are 'smooth' and that there are no costs involved in switching from the one technique to another. Nor are there any costs involved in switching from the one technology to another. A further assumption is that every single producer invests in the newest technology only.[2] At the same time we assume that capital costs are sunk costs ex-post.

Because of the smooth substitution possibilities ex-post, it follows that output can be produced using all the technologies which have come into existence from time immemorial. The reason is that it is possible to increase the marginal productivity of labour (and thus decrease variable costs per unit of output) indefinitely for any production function which obeys the Inada conditions. The practical importance of this phenomenon will become more clear below.

We furthermore assume that entrepreneurs are price-takers on the factor markets as well as on the output market. Moreover, we assume that producers form expectations using a partial adjustment scheme regarding expected rates of growth. These expectations are important considering the fact that, given the (more) limited substitution possibilities ex-post, entrepreneurs can only try to avoid the cost-consequences of a change in future wage-rates in as far as these latter changes have been anticipated.

2.2 The Production Technology

For the ex-ante production function as well as the ex-post production functions we use linear homogeneous CES functions. With respect to vintage i at time t, we denote the level of capacity output, the level of labour demand at full capacity operation and the level of investment by $Y_{i,t}$, $N_{i,t}$ and $I_{i,t}$, respectively. We therefore have :

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2 Meijers (1994) uses the notion of lags in the adoption of new technologies in order to account for diffusion aspects of the transmission of technological change in a vintage setting. However, the diffusion approach requires a considerable increase in the 'vintage book-keeping' overhead, and it is therefore not pursued here.

$$Y_{t,t} = \left\{A_t^a \cdot (N_{t,t})^{-\rho_a} + B_t^a \cdot (I_{t,t})^{-\rho_a}\right\}^{-1/\rho_a} \tag{1}$$

where the super-/subscript a denotes the ex-ante function. A^a_t and B^a_t are the CES distribution parameters and $\sigma_a = 1/(1+\rho_a)$ is the ex-ante elasticity of substitution. Similarly, for the ex-post production function we have :

$$Y_{i,t} = \left\{A_{i,t}^p \cdot (N_{i,t})^{-\rho_p} + B_{i,t}^p \cdot (I_{i,t})^{-\rho_p}\right\}^{-1/\rho_p} \tag{2}$$

where t>i, and where the super-/subscript p denotes the ex-post parameters. Note that the main difference between (1) and (2) lies in the specification of the distribution parameters A and B. In the ex-ante case A depends on the time of installation only (embodied technical change stops at the moment of installation), whereas in the ex-post case A and B depend also on the time of observation: disembodied technical change takes over from embodied technical change (with a possibly different rate) from the moment of installation of a piece of equipment.

With respect to the ex-ante and ex-post distribution parameters we subsume the influence of a change in working hours as well as disembodied technical change under the distribution parameters themselves. Hence :

$$A_t^a = A_0 \cdot (1+\mu_n)^{-\rho_a \cdot t} \cdot h_t^{-\rho_a \cdot \varepsilon_n} \tag{3}$$

$$B_t^a = B_0 \cdot (1+\mu_I)^{-\rho_a \cdot t} \cdot h_t^{-\rho_a \cdot \varepsilon_I}$$

$$A_{i,t}^p = A_{i,i}^p \cdot (1+\gamma_n)^{-\rho_p \cdot (t-i)} \cdot \left\{\frac{h_t}{h_i}\right\}^{-\rho_p \cdot \varepsilon_n}$$

$$B_{i,t}^p = B_{i,i}^p \cdot (1+\gamma_I)^{-\rho_p \cdot (t-i)} \cdot \left\{\frac{h_t}{h_i}\right\}^{-\rho_p \cdot \varepsilon_n}$$

where μ_n and μ_I are the rates of embodied labour and capital augmenting technological change, respectively. γ_n and γ_I are their disembodied technological change equivalents. h_t is the index of working hours, while the ε 's are the working hours elasticities of effective labour and capital input. The ε 's have been set equal to 0.75. [3] It should be noted that the ex-post distribution parameters still need to be linked to their ex-ante counterparts. This will be done in sections 2.3 and 2.4 below.

2.3 Optimising Behaviour and Myopic Foresight

In Figure 1 below, the ex-ante unit iso-quant has been labelled e.a., while two ex-post iso-quants have been labelled e.p. The ex-ante iso-quant has been drawn as an envelope of all possible ex-post iso-quants. We have also drawn two different wage / rental ratios, which give rise to two different optimum values of the labour intensity of production on new (and old) equipment.

Suppose now that at time 0 the ruling wage rental ratio is such that point A would be chosen. Then, at time 1, the wage rental ratio changes such that on new equipment point B becomes optimum. With the rise in the relative wage rate, the labour / capital ratio on new equipment has a tendency to fall. But on old equipment, substitution possibilities between labour and capital are more limited by assumption, and therefore the rise in the relative wage rate invokes only a moderate adjustment of the labour / capital ratio on old equipment. This is depicted by the move from point A to point C along the ex-post iso-quant, as opposed to the 'move' from point A to point B along the ex-ante iso-quant. Obviously, when substitution possibilities ex-post would be equal to those ex-ante, the distinction between old equipment and new equipment vanishes entirely in the absence of embodied technological change.

Note that by choosing a specific technique on one of the infinitely many ex-post iso-quants which are associated with a certain ex-ante technology, one also chooses one's future substitution possibilities with respect to the vintage under consideration. Hence, when one would expect the 'average future relative wage rate' to be given by the slope of the straight line through B, while the initial relative wage rate is given by slope of the straight line through A, one would probably do better choosing the ex-post iso-quant implicitly defined by point B and with point D as the 'entry technique', rather than the one which is implicitly defined by point A. We will come back to this in more detail below.

3 See also den Hartog and Tjan (1976), Kuipers and van Zon (1982) and Muysken and van Zon (1987), who have used a similar approach.

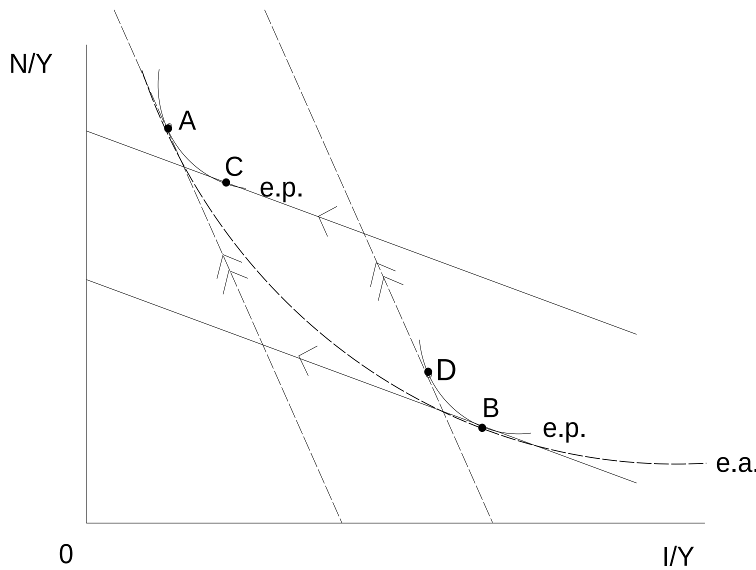

Fig. 1 The ex-ante production function envelope and myopic foresight

2.4 Linking Ex-Ante and Ex-Post Substitution Characteristics

Figure 1 shows that two conditions should hold in order for the ex-post iso-quant in question to be consistent with the ex-ante choice set :

1 the tangential technique (denoted by $(\bar{\nu}, \bar{\kappa})$, which are the labour / output ratio and the capital / output ratio, respectively) should be part of the ex-ante envelope as well as part of its associated ex-post unit iso-quant ;

2 since the envelope has only one technique in common with each ex-post unit iso-quant, and since substitution possibilities ex-ante and ex-post are 'smooth' by assumption, it follows that for the tangential technique $(\bar{\nu}, \bar{\kappa})$ the slopes of both the ex-ante iso-quant and the ex-post iso-quant should be the same.

From requirement 2 it follows for the tangential technique $(\bar{\nu}, \bar{\kappa})$ that :

$$\left(\frac{d\bar{\nu}}{d\bar{\bar{\kappa}}}\right)^{ex\ ante} = \left(\frac{d\bar{\nu}}{d\bar{\bar{\kappa}}}\right)^{ex\ post} \Rightarrow \frac{B_t^a}{A_t^a} \cdot \left\{\frac{\bar{\kappa}}{\bar{\bar{\nu}}}\right\}^{-1/\sigma_a} = \frac{B_{t,t}^p}{A_{t,t}^p} \cdot \left\{\frac{\bar{\kappa}}{\bar{\bar{\nu}}}\right\}^{-1/\sigma_p} \Rightarrow A_{t,t}^p = \frac{B_{t,t}^p}{B_t^a} \cdot A_t^a \cdot \left(\frac{\bar{\nu}}{\bar{\bar{\kappa}}}\right)^{\rho_p - \rho_a} \tag{4}$$

From requirement 1 it follows moreover that the labour coefficient ex-post as well as the capital coefficient ex-post should be equal to their ex-ante counterparts. Hence, for the tangential technique $(\bar{\nu}, \bar{\kappa})$ we should have : [4]

4 Note that we have assumed the ex-ante and ex-post CES functions to be linear homogeneous.

$$A^p_{t,t} \cdot \bar{\nu}^{-\rho_p} + B^p_{t,t} \cdot \bar{\kappa}^{-\rho_p} = A^a_t \cdot \bar{\nu}^{-\rho_a} + B^a_t \cdot \bar{\kappa}^{-\rho_a} = 1 \tag{5}$$

Substitution of (4) into (5) yields :

$$B^p_{t,t} = B^a_t \cdot \bar{\kappa}^{-(\rho_p - \rho_a)} \tag{6}$$

Furthermore, substitution of (6) into (4) yields in turn :

$$A^p_{t,t} = A^a_t \cdot \bar{\nu}^{-(\rho_p - \rho_a)} \tag{7}$$

Equations (6) and (7) show that the ex-post distribution parameters are uniquely determined by the tangential technique $(\bar{\nu}, \bar{\kappa})$.[5] Hence, by choosing a tangential technique $(\bar{\nu}, \bar{\kappa})$, one also chooses a unique ex-post unit iso-quant in the process. We will now introduce equations (6) and (7) in an intertemporal optimization setting where one not only has to select the optimum initial factor-coefficients ex-ante and ex-post, but also the appropriate tangential techniques $(\bar{\nu}, \bar{\kappa})$.

2.5 Factor Demand, Limited Substitution Possibilities Ex-Post and Expectations

2.5.1 Introduction

We assume that entrepreneurs try to maximise the net present value of their productive activities now and in the future by :

1 allocating labour in the ’right’ way among existing vintages and new vintages;

2 allocating labour and capital in the ’right’ way to new vintages;

3 determining the production shares of new and old vintages.

The way in which entrepreneurs can try to achieve this goal, is **first** by selecting the capital/output and labour/output ratios on existing as well as new vintages in accordance with (their expectations regarding) relative prices, **secondly** by selecting ’optimum’ ex-post production technologies for new equipment, and **third** by selecting the volume of investment in new equipment. These choices are all conditional on an aggregate capacity output constraint (which may be based on expectations regarding

5 Note that embodied technological change has an influence on the ex-post CES distribution parameters through their dependence on the ex-ante parameters. Note also that, in the case of identical elasticities of substitution ex-ante and ex-post, the ex-post CES distribution parameters are identical to their ex-ante counterparts.

demand, which are however not specified any further here). They are also conditional on the functional forms of the ex-post production functions and of the ex-ante production function, the latter of which defines the infinitely large family of ex-post isoquants from which entrepreneurs will have to choose a single ex-post iso-quant.

2.5.2 Choosing Optimum Factor Proportions

We assume input-prices to be given to individual entrepreneurs, while output prices are assumed exogenously given. Moreover, w_t is the current wage rate, and q_t is the cost of a unit of capital. X_t is the total amount of output to be produced on both new equipment and old equipment. δ is the technical decay parameter (we assume depreciation by 'radioactive decay').

The expected present value of the firm is then given by :

$$\begin{aligned}
\Phi_t = & \sum_{j=-\infty}^{t-1} \sum_{i=t}^{\infty} p'_i \cdot \frac{I_{j,j} \cdot (1-\delta)^{i-j}}{\kappa_{j,i}} \cdot \left(1 - \frac{w'_i}{p'_i} \cdot \nu_{j,i}\right) \\
& + \sum_{j=t}^{\infty} \sum_{i=j}^{\infty} p'_i \cdot \frac{I_{j,j} \cdot (1-\delta)^{i-j}}{\kappa_{j,i}} \cdot \left(1 - \frac{w'_i}{p'_i} \cdot \nu_{j,i}\right) \\
& - \sum_{j=t}^{\infty} q'_j \cdot I_{j,j} \\
& + \sum_{j=-\infty}^{t-1} \sum_{i=t}^{\infty} \lambda^o_{j,i} \cdot (f^{j,i}(\nu_{j,i}, \kappa_{j,i}) - 1) \\
& + \sum_{j=t}^{\infty} \sum_{i=j}^{\infty} \lambda^n_{j,i} \cdot (f^{j,i}(\nu_{j,i}, \kappa_{j,i}, \bar{\nu}_j, \bar{\kappa}_j) - 1) \\
& + \sum_{i=t}^{\infty} \lambda^x_i \cdot \left(X_i - \sum_{j=-\infty}^{i} \frac{I_{j,j} \cdot (1-\delta)^{i-j}}{\kappa_{j,i}}\right) \\
& + \sum_{j=t}^{\infty} \lambda^g_j \cdot (1 - g^j(\bar{\nu}_j, \bar{\kappa}_j))
\end{aligned} \tag{8}$$

where p', w' and q' are the expected present value for time t of the price of a unit of output, of a unit of labour and of a unit of investment, respectively, and where $p'_i = p_i \cdot (1 + r_t)^{-(i-t)}$. w' and q' are defined similarly. $f^{j,i}()$ is the linear homogeneous ex-post production function associated with vintage j at time i, while $g^j()$ is the linear homogeneous ex-ante production function associated with vintage j at the time of its installation. Both $f^{j,i}()$ and $g^j()$ have been defined in terms of 'technical coefficients' rather than in terms of the absolute factor-inputs. The time index t represents the present (decision) moment, while the index j is associated with the time of installation of a specific vintage. Furthermore, i is a time index which refers to the present and the future. $\lambda^o, \lambda^n, \lambda^x$ and λ^g denote the Lagrange multipliers associated with the ex-post production function constraints for old and new vintages, the capacity requirement constraint and the ex-ante production function constraint, respectively.

The first term of Φ_t represents the expected present value of all the quasi-rents associated with the operation of the old vintages which were installed up to and including time t-1. The only variables under the control of producers in this case are the labour / output and capital / output ratios, since the amount of investment and the nature of investment (in terms of the (endogenous) distribution parameters of the associated ex-post production function) of these vintages have already been determined in the past. The second and third term taken together represent the expected net present value of the rents to be earned on the new vintages which will be installed from (and including) time t. Hence :

$$\frac{\partial \Phi_t}{\partial \nu_{j,i}} = -w'_i \cdot Y_{j,i} + \lambda^o_{j,i} \cdot \frac{\partial f^{j,i}}{\partial \nu_{j,i}} = 0 \qquad \forall j < t, i \geq t \tag{9.A}$$

$$\frac{\partial \Phi_t}{\partial \nu_{j,i}} = -w'_i \cdot Y_{j,i} + \lambda^n_{j,i} \cdot \frac{\partial f^{j,i}}{\partial \nu_{j,i}} = 0 \qquad \forall j \geq t, i \geq j \tag{9.B}$$

$$\frac{\partial \Phi_t}{\partial \kappa_{j,i}} = -p'_i \cdot \frac{Y_{j,i}}{\kappa_{j,i}} \cdot \left(1 - \frac{w'_i}{p'_i} \cdot \nu_{j,i}\right) + \lambda^o_{j,i} \cdot \frac{\partial f^{j,i}}{\partial \kappa_{j,i}} + \lambda^x_i \cdot \frac{Y_{j,i}}{\kappa_{j,i}} = 0 \qquad \forall j < t, i \geq t \tag{9.C}$$

$$\frac{\partial \Phi_t}{\partial \kappa_{j,i}} = -p'_i \cdot \frac{Y_{j,i}}{\kappa_{j,i}} \cdot \left(1 - \frac{w'_i}{p'_i} \cdot \nu_{j,i}\right) + \lambda^n_{j,i} \cdot \frac{\partial f^{j,i}}{\partial \kappa_{j,i}} + \lambda^x_i \cdot \frac{Y_{j,i}}{\kappa_{j,i}} = 0 \qquad \forall j \geq t, i \geq j \tag{9.D}$$

$$\frac{\partial \Phi_t}{\partial I_{j,j}} = \sum_{i=j}^{\infty} p'_i \cdot \frac{(1-\delta)^{i-j}}{\kappa_{j,i}} \cdot \left(1 - \frac{w'_i}{p'_i} \cdot \nu_{j,i}\right) - q'_j - \sum_{i=j}^{\infty} \frac{(1-\delta)^{i-j}}{\kappa_{j,i}} \cdot \lambda^x_i = 0 \qquad \forall j \geq t, i \geq j \tag{9.E}$$

$$\frac{\partial \Phi_t}{\partial \bar{\nu}_j} = \sum_{i=j}^{\infty} \frac{\partial f^{j,i}}{\partial \bar{\nu}_j} \cdot \lambda^n_{j,i} - \lambda^g_j \cdot \frac{\partial g^j}{\partial \bar{\nu}_j} = 0 \qquad \forall j \geq t, i \geq j \tag{9.F}$$

$$\frac{\partial \Phi_t}{\partial \bar{\kappa}_j} = \sum_{i=j}^{\infty} \frac{\partial f^{j,i}}{\partial \bar{\kappa}_j} \cdot \lambda^n_{j,i} - \lambda^g_j \cdot \frac{\partial g^j}{\partial \bar{\kappa}_j} = 0 \qquad \forall j \geq t, i \geq j \tag{9.G}$$

where $Y_{j,i}$ is defined as the output associated with vintage j at time i, i.e. :

$$Y_{j,i} = \frac{I_{j,j} \cdot (1-\delta)^{i-j}}{\kappa_{j,i}} \qquad \forall i \geq j \tag{10}$$

Because of the linear homogeneity of $f^{j,i}()$ in $\nu_{j,i}$ and $\kappa_{j,i}$, it follows from the application of the Euler-equation to equations (9.A) and (9.C) that :

$$\lambda^{o}_{j,i} = (p'_i - \lambda^x_i) \cdot Y_{j,i} \qquad \forall\, j < t, i \geq t \tag{11}$$

Likewise, from (9.B) and (9.D) it follows that :

$$\lambda^{n}_{j,i} = (p'_i - \lambda^x_i) \cdot Y_{j,i} \qquad \forall\, j \geq t, i \geq j \tag{12}$$

Substitution of equations (11) and (12) into (9.A) and (9.B), leads to the conclusion that all marginal labour productivities should be equal, both for existing vintages and for vintages still to be installed, since the ratio $w'_i/(p'_i - \lambda^x_i)$ is independent of the time of installation of a particular vintage.

It is relatively easy to obtain the value of $p'_j - \lambda^x_j$: [6]

$$p'_j - \lambda^x_j = q'_j \cdot \kappa_{j,j} \cdot \left(1 - \frac{(1-\delta)\cdot(1+\hat{q}_j)}{1+r_j}\right) + w'_j \cdot \nu_{j,j} \tag{13}$$

which, when substituted into (9.B) and (9.D) for i=j, gives :

$$\left(\frac{\partial f^{j,j}}{\partial \nu_{j,j}}\right) \Big/ \left(\frac{\partial f^{j,j}}{\partial \kappa_{j,j}}\right) = \frac{w'_j}{q'_j \cdot \left(1 - \frac{(1-\delta)\cdot(1+\hat{q}_j)}{1+r_j}\right)} = \frac{w'_j}{q''_j} \tag{14}$$

where q''_j is implicitly defined by (14) and where we have substituted equations (13), (9.B) and (9.D), and where $\hat{q}$ represents the (expected) rate of growth of the price-index of investment goods.[7]

Note that equations (9.A), (11) and (13) imply that:

$$p'_i - \lambda^x_i = \frac{w'_i \cdot \Delta N_{j,i}}{\frac{\partial f^{j,i}}{\partial \nu_{j,i}} \cdot \Delta N_{j,i}} = q'_i \cdot \kappa_{i,i} \cdot \left(1 - \frac{(1-\delta)\cdot(1+\hat{q}_i)}{1+r_i}\right) + w'_i \cdot \nu_{i,i} \tag{15}$$

6 See Appendix A.

7 Note that q''_t is approximately equal to the user cost of capital as it is usually defined, i.e. $q''_t = q_t.(r_t + \delta - \hat{q})$.

Where the middle part of equation (15) represents the (variable) costs of the marginal unit of labour per unit of (marginal) output. $\Delta N_{j,i}$ represents the marginal unit of labour to be allocated to vintage j at time i.

Equation (15) essentially says that the marginal unit of output produced on old equipment should be produced at a variable cost which is equal to the unit total cost of output on new equipment. This is exactly what the Malcomson scrapping condition says with regard to the optimum vintage composition of the capital stock in a situation of cost-minimisation (c.f. Malcomson (1975)).

From (14) it follows immediately that :

$$\frac{A^p_{j,j}\cdot \nu_{j,j}^{-(1+\rho_p)}}{B^p_{j,j}\cdot \kappa_{j,j}^{-(1+\rho_p)}}=\frac{w'_j}{q''_j} \quad\Rightarrow\quad \nu_{j,j}=\kappa_{j,j}\cdot\left(\frac{B^p_{j,j}\cdot w'_j}{A^p_{j,j}\cdot q''_j}\right)^{-1/(1+\rho_p)}=h_j\cdot\kappa_{j,j} \tag{16}$$

where h_j is implicitly defined by (16). Substitution of (16) into the ex-post production function $f^{j,j}()$ yields therefore :

$$\kappa_{j,j}=\left\{A^p_{j,j}\cdot h_j^{-\rho_p}+B^p_{j,j}\right\}^{1/\rho_p} \tag{17}$$

$$\nu_{j,j}=\left\{A^p_{j,j}+B^p_{j,j}\cdot h_j^{\rho_p}\right\}^{1/\rho_p}$$

where $A^p_{j,j}$ and $B^p_{j,j}$ depend on the tangential technique to be chosen from the ex-ante function.

2.5.3 Choosing the Optimum Ex-Post Iso-Quant

In a putty-clay vintage model of production, it is possible to ’condense’ information about the future into a present-value price system which is used to select the optimum ex-post iso-quant and the optimum entry point of the new technique on that iso-quant at the same time (see Meijers and van Zon (1991) for an explicit account of such a procedure in a multi-level CES dual cost-function setting, as well as Kuipers and van Zon (1982) and Muysken and van Zon (1987) for less explicit applications of the present value price system). The reason is simply that, in the case of a Leontief ex-post production function, the entry point must be somewhere on the ex-ante iso-quant and it will stay at that position indefinitely, except for the influence of disembodied technical change. In a putty-semi-putty situation, however, labour coefficients ex-post can vary in response to changes in relative prices too. Moreover, given the optimality condition that at any point in time all marginal labour productivities should be equal, the future time-path of wage costs associated with a specific technique chosen today, is also influenced by what future technologies will look like in terms of their technical characteristics.[8] Hence, in this case, the impact of future circumstances on current decisions can not be condensed that easily into a 'pure' present value price-system. Rather, the future needs to be integrated into the decision framework in a somewhat different way. In this context, it should be noted that equations (9.F) and (9.G) describe the marginal conditions which the tangential techniques have to obey in order to ensure that the net present value of the firm is maximised, also in an intertemporal setting.

Using equations (3),(5), (6), (9.F) and (9.G), we have :

$$\frac{\partial f^{j,i}}{\partial \bar{\nu}_j} = -\left(\frac{1}{\rho_p}\right)\cdot\frac{\partial A^p_{j,i}}{\partial \bar{\nu}_j} = -\left(\frac{\rho_p-\rho_a}{\rho_p}\right)\cdot\frac{A^p_{j,i}}{\bar{\nu}_j}\cdot \nu_{j,i}^{-\rho_p} = -\left(\frac{\rho_p-\rho_a}{\rho_p}\right)\cdot\frac{\partial f^{j,i}}{\partial \nu_{j,i}}\cdot\frac{\nu_{j,i}}{\bar{\nu}_j} \tag{18}$$

$$\frac{\partial f^{j,i}}{\partial \bar{\kappa}_j} = -\left(\frac{1}{\rho_p}\right)\cdot\frac{\partial B^p_{j,i}}{\partial \bar{\kappa}_j} = -\left(\frac{\rho_p-\rho_a}{\rho_p}\right)\cdot\frac{B^p_{j,i}}{\bar{\kappa}_j}\cdot \nu_{j,i}^{-\rho_p} = -\left(\frac{\rho_p-\rho_a}{\rho_p}\right)\cdot\frac{\partial f^{j,i}}{\partial \kappa_{j,i}}\cdot\frac{\kappa_{j,i}}{\bar{\kappa}_j}$$

Using (18) and the linear homogeneity of $f^{j,i}()$ and $g^j()$, and applying the Euler equation to (9.F) and (9.G), we obtain :

$$\begin{aligned}\lambda^g_j &= \lambda^g_j\cdot\left(\frac{\partial g^j}{\partial \bar{\nu}_j}\cdot\bar{\nu}_j + \frac{\partial g^j}{\partial \bar{\kappa}_j}\cdot\bar{\kappa}_j\right) = \sum_{i=j}^{\infty}\lambda^n_{j,i}\cdot\left(\frac{\partial f^{j,i}}{\partial \bar{\nu}_j}\cdot\bar{\nu}_j + \frac{\partial f^{j,i}}{\partial \bar{\kappa}_j}\cdot\bar{\kappa}_j\right) \\ &= -\left(\frac{\rho_p-\rho_a}{\rho_p}\right)\cdot\sum_{i=j}^{\infty}\lambda^n_{j,i}\cdot\left(\frac{\partial f^{j,i}}{\partial \nu_{j,i}}\cdot\nu_{j,i} + \frac{\partial f^{j,i}}{\partial \kappa_{j,i}}\cdot\kappa_{j,i}\right) = -\left(\frac{\rho_p-\rho_a}{\rho_p}\right)\cdot\sum_{i=j}^{\infty}\lambda^n_{j,i}\end{aligned} \tag{19}$$

Using (18), (19), (9.B) and (9.E) it follows that :

$$\frac{\partial g^j}{\partial \bar{\nu}_j}\cdot\bar{\nu}_j = \frac{\sum_{i=j}^{\infty}\frac{\partial f^{j,i}}{\partial \nu_{j,i}}\cdot\nu_{j,i}\cdot\lambda^n_{j,i}}{\sum_{i=j}^{\infty}\lambda^n_{j,i}} = \frac{\sum_{i=j}^{\infty}w'_i\cdot\nu_{j,i}\cdot Y_{j,i}}{\sum_{i=j}^{\infty}(p'_i-\lambda^x_i)\cdot Y_{j,i}} = \frac{\sum_{i=j}^{\infty}w'_i\cdot\nu_{j,i}\cdot(1-\delta)^{i-j}}{\sum_{i=j}^{\infty}(p'_i-\lambda^x_i)\cdot(1-\delta)^{i-j}} \tag{20}$$

which is equal to the output weighted average of the present value of unit labour costs over unit total costs on the newest vintage. The rightmost part of (20) rests on the assumption that the rate of decrease of $Y_{j,i}$ can be (roughly) approximated by the value

8 Note that such a dependence on future technological characteristics of current technological choices is also implied by the Malcomson scrapping condition in a full putty-clay vintage setting.

of the decay parameter. [9] Note, however, that since the relative size of the error is the same in both the nominator and the denominator of (20), the errors cancel each other to some extent.

From equations (9.A), (11) and (18) we immediately obtain :

$$\nu_{j,i} = \left(\frac{w'_i}{A^p_{j,i} \cdot (p'_i - \lambda^x_i)} \right)^{-\sigma_p} = (A^p_{j,i})^{\sigma_p} \cdot \left(\frac{w'_i}{\psi'_i} \right)^{-\sigma_p} \tag{21}$$

where ψ'_i is implicitly defined by (21) and equal to the expected present value of future unit total costs on the newest vintage. Substitution of (21) into (20) yields therefore :

$$\frac{\partial g^t}{\partial \bar{\nu}_t} \cdot \bar{\nu}_t = \frac{\sum_{i=t}^{\infty} (A^p_{t,i})^{\sigma_p} \cdot (w'_i)^{1-\sigma_p} \cdot \psi_i^{\sigma_p} \cdot (1-\delta)^{i-t}}{\sum_{i=t}^{\infty} \psi'_i \cdot (1-\delta)^{i-t}} \tag{22}$$

Assuming constant rates of growth of the variables in (22), where applicable, we immediately obtain :

$$\frac{\partial g^t}{\partial \bar{\nu}_t} \cdot \bar{\nu}_t = \frac{(A^p_{t,t})^{\sigma_p} \cdot \left(\frac{w'_t}{\psi_t} \right)^{1-\sigma_p} \cdot \sum_{i=t}^{\infty} \left\{ (1+\hat{A}^p_{t,t})^{\sigma_p} \cdot (1+\hat{w}'_t)^{1-\sigma_p} \cdot (1+\hat{\psi}'_t)^{\sigma_p} \cdot (1-\delta) \right\}^{i-t}}{\sum_{i=t}^{\infty} \{ (1+\hat{\psi}'_t) \cdot (1-\delta) \}^{i-t}} \tag{23}$$

Because $A^p_{t,i}$ depends explicitly on the 'tangential technique', it follows that (23) can be rewritten as :[10]

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9 Note that this is indeed a rough approximation, since the latter also assumes that the amount of labour allocated to the machinery in question would also have to fall at a rate equal to δ, since otherwise output could not fall at that rate (the ex-post production function is linear homogeneous by assumption). However, when wage costs on an old vintage rise more rapidly than average total costs on a new vintage, then marginal labour productivity on the old vintage should rise in compensation, and hence labour input should fall more rapidly than capital input. The ensuing rate of decrease of output would be somewhere in between the different rates of decrease of both inputs.

10 Of course, ψ_t also depends on the tangential technique, but at this stage we ignore this, since the latter dependency is a more implicit one, and must be taken account of during the simultaneous solution of the model itself.

$$A^{a}_{t,t} \cdot \bar{\nu}_t^{-\rho_a} = (A^{p}_{t,t})^{\sigma_p} \cdot Z_t = A^{a\ \sigma_p}_{t,t} \cdot \bar{\nu}_t^{-(\rho_p - \rho_a) \cdot \sigma_p} \cdot Z_t \tag{24}$$

where Z_t is a collection of terms implicitly defined by (23) and (24) taken together. Hence, from (24) we immediately obtain :

$$\bar{\nu}_t = \left\{ A^{a}_{t,t} \cdot Z_t^{-\frac{1}{1-\sigma_p}} \right\}^{\sigma_a} \tag{25}$$

As long as the growth of nominal wages and the growth of the price of investment is at most equal to the rate of interest, both the summations present in equation (25) have a finite value, because the multiplicative term in the geometric expansion is less than one. But in practice, we will not assume an infinitely long planning horizon : we stick to the findings of Kuipers and van Zon (1982), Gelauff, Wennekers and de Jong (1985) and Muysken and van Zon (1987), who find (on average) a planning period of about 15 years. Hence, redefining Z_t to be equal to the summation of the geometric expansion of (23) over the first Θ years (where we take Θ to be equal to 15 during the estimation of the model), we have :

$$Z_t = \left(\frac{w_t}{\psi_t} \right)^{1-\sigma_p} \cdot \frac{S\left(t, \left\{(1+\gamma_n) \cdot (1+\hat{h}_t)^{\varepsilon_n}\right\}^{-\rho_p \cdot \sigma_p} \cdot (1+\hat{w}_t)^{1-\sigma_p} \cdot (1+r_t)^{-1} \cdot (1+\hat{\psi})^{\sigma_p} \cdot (1-\delta), \Theta\right)}{S(t, 1, (1+\hat{\psi}_t) \cdot (1-\delta)/(1+r_t), \Theta)} \tag{26}$$

where

$$S(t,q,\Theta) = 1 + q_t + q_t^2 + .. + q_t^{\Theta} = \frac{(1-q_t^{\Theta+1})}{1-q_t}$$

and where $\hat{\psi}_t$ stands for the expected rate of growth of undiscounted unit production cost on the newest vintage.

A minor problem still remains to be resolved : the rate of growth of future unit production costs is not known. In order to avoid the computational implications and complications of fully forward looking behaviour, we will use the four year moving average of the 'realised' rate of growth of unit production cost on the newest vintage.

[11] Then, given $\hat{\psi}$, the optimum allocation of labour to existing vintages can be described using (21). And so, given the ex-post production functions and the existing capital stock, it is possible to obtain total capacity output associated with the existing capital stock, after which the capacity gap to be filled by output from the newest equipment can be obtained. Given the optimum capital coefficient for new equipment, the amount of investment follows directly from the size of the capacity gap, and so does the required amount of labour associated with the newest vintage. Thus, we are able to arrive at aggregate capacity labour demand and aggregate 'capital' demand by summing over all vintages which are in existence at some moment of time. **The problem is that there are infinitely many vintages.** Hence, adding them all together in order to obtain aggregate capacity output and aggregate capacity labour demand is simply not possible. We therefore present a **practical shortcut** in the next section.

2.6 The Recursive Update Rules

From the first order conditions for a profit maximum (c.f. (9.A), (9.B), (11) and (12)), it follows that all marginal labour productivities should be the same for existing machinery and equipment and for new machinery. Using (9) we therefore have :

$$A^p_{t,t} \cdot \{\nu_{t,t}\}^{-1/\sigma_p} = A^p_{i,t} \cdot \{\nu_{i,t}\}^{-1/\sigma_p} \quad \Rightarrow \quad \nu_{i,t} = \nu_{t,t} \cdot (A^p_{t,t})^{-\sigma_p} \cdot (A^p_{i,t})^{\sigma_p} = \xi_t \cdot (A^p_{i,t})^{\sigma_p} \tag{27}$$

where ξ_t is implicitly defined by (27). (27) shows that the optimum value of the labour coefficient on an existing vintage consists of a vintage specific part and a general part. The corresponding value of the capital coefficient can be obtained from the ex-post production function :

$$\{\kappa_{i,t}\}^{-\rho_p} = \frac{1}{B^p_{i,t}} - \frac{A^p_{i,t} \cdot \{\nu_{i,t}\}^{-\rho_p}}{B^p_{i,t}} \tag{28}$$

Using (27) and (28), we immediately obtain :

$$\zeta_{i,t} = \left\{\frac{1}{\kappa_{i,t}}\right\}^{\rho_p} = \frac{1}{B^p_{i,t}} - (\xi_t)^{-\rho_p} \cdot \frac{(A^p_{i,t})^{\sigma_p}}{B^p_{i,t}} \tag{29}$$

11 We also took the four year moving average of the actual growth of wages and of working hours to represent their expected growth rates. With respect to the discount rate, we assumed it to be equal to the sum of the four year moving average of the yield on government bonds and a constant sector-specific risk-premium which is one of the 'parameters' to be estimated later on.

Equation (29) provides one of the central equations of the RUM model. Note that $\zeta_{i,t}$ is implicitly defined as the capital productivity of vintage i at time t, raised to the power of ρ_p. Let us now define :

$$\bar{\zeta}_t = \sum_{i=-\infty}^{t} \zeta_{i,t} \cdot \frac{I_{i,t}}{\sum_{j=-\infty}^{t} I_{j,t}} = \sum_{i=-\infty}^{t} \zeta_{i,t} \cdot \frac{I_{i,t}}{K_t} = \sum_{i=-\infty}^{t} \zeta_{i,t} \cdot S_{i,t} \tag{30}$$

where $S_{i,t}$ is the volume share of investment at time i in the capital stock at time t. $\bar{\zeta}_t$ is a weighted average of all individual 'capital productivities' of the separate vintages with the investment shares in the total capital stock as weights. Note that when ρ_p is equal to 1, i.e. the ex-post elasticity of substitution is equal to 0.5, then (31) provides the 'exact' value of the aggregate capital productivity. When ρ_p is not equal to 1, we approximate the average capital productivity (π_t) by:

$$\pi_t = \{\bar{\zeta}_t\}^{1/\rho_p} \tag{31}$$

Note that (31) implies that for ρ_p not equal to 1 the aggregate productivity of capital is obtained as a 'CES average' of the individual capital productivities at the vintage level, since (29), (30) and (31) taken together imply :

$$\pi_t = \left\{ \sum_{i=-\infty}^{t} S_{i,t} \cdot \left(\frac{1}{\kappa_{i,t}} \right)^{\rho_p} \right\}^{1/\rho_p} \tag{32}$$

Equation (32) provides another approximation which is needed to define the RUM model.[12] Using (29) and (30) we obtain:

$$\bar{\zeta}_t = \sum_{i=-\infty}^{t} \frac{S_{i,t}}{B^p_{i,t}} - (\xi_t)^{-\rho_p} \cdot \sum_{i=-\infty}^{t} S_{i,t} \cdot \frac{(A^p_{i,t})^{\sigma_p}}{B^p_{i,t}} \tag{33}$$

Equation (33) can be re-defined as :

$$\bar{\zeta}_t = T_{1,t} - (\xi_t)^{-\rho_p} \cdot T_{2,t} \tag{34}$$

12 In various simulation experiments described in van Zon (1994) it is shown that the approximation is indeed a good one.

Note that in the absence of disembodied technical change, the two ways in which $T_{1,t}$ and $T_{2,t}$ depend on time are first through $S_{i,t}$ and secondly through the upper limits of the respective summations. For $T_{1,t}$ we conclude therefore that its value must be equal to $T_{1,t-1}$ except for the fact that the overall weight of already existing vintages in the determination of $T_{1,t}$ must have decreased when gross investment is positive, while on the other hand the relative weights $S_{i,t} / S_{j,t}$ for i,j<t are not changed at all since technical decay takes place at a constant rate. Therefore, the transition from t-1 to t implies that the weight of existing machinery (i.e. the machinery installed up to and including time t-1) in the determination of the average value of capital productivity at time t has become $(1-\delta) \cdot K_{t-1}/K_t$, whereas the weight of the capital productivity of the new vintage in aggregate capital productivity is equal to $I_{t,t}/K_t$. A similar reasoning holds for the change in the value of $T_{2,t}$.

With regard to disembodied technical change, it should be noted that (by assumption) it affects existing vintages only. Moreover, it affects those vintages to the same extent. Hence, disembodied technical change (as well as the influence of a change in working hours) can be introduced into the model quite easily by defining the terms T1,t and T2,t as follows: [13]

$$T_{1,t} = T_{1,t-1} \cdot \frac{(1-\delta) \cdot K_{t-1}}{K_t} \cdot \left((1+\gamma_I) \cdot (1+\hat{h}_t)^{\varepsilon_I}\right)^{\rho_p} + \left(\frac{1}{B_{t,t}^p}\right) \cdot \frac{I_{t,t}}{K_t} \tag{35}$$

$$T_{2,t} = T_{2,t-1} \cdot \frac{(1-\delta) \cdot K_{t-1}}{K_t} \cdot \left(\frac{\left((1+\gamma_n) \cdot (1+\hat{h}_t)^{\varepsilon_n}\right)^{\sigma_p}}{(1+\gamma_I) \cdot (1+\hat{h}_t)^{\varepsilon_I}}\right)^{-\rho_p} + \left(\frac{(A_{t,t}^p)^{\sigma_p}}{B_{t,t}^p}\right) \cdot \frac{I_{t,t}}{K_t}$$

$$K_t = (1-\delta) \cdot K_{t-1} + I_{t,t}$$

$$\pi_t = \left\{T_{1,t} - \xi_t^{-\rho_p} \cdot T_{2,t}\right\}^{1/\rho_p}$$

Equation (35) now shows that the capital productivity 'book-keeping' of an infinitely large family of vintages can be reduced to a fairly small set of equations. [14] Moreover,

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13 This way of handling disembodied technical change follows directly from the fact that an expression $X_t = X_0 \cdot (1+x)^{\beta \cdot t}$ can be written as : $X_t = X_{t-1} \cdot (1+x)^{\beta}$. Hence, X_t can be obtained by 'updating' X_{t-1} by means of the factor $(1+x)^{\beta}$. See also equation (3).

14 Note that a related approach is described in Eigenraam (1987), although the link between productivity aggregates and the development of the capital stock is less direct there than in the RUM model.

equation (35) shows that the value of aggregate capital productivity can be obtained by means of a (time-) recursive update of its composing terms, rather than by explicitly obtaining it from the underlying individual vintages.

With regard to the determination of the aggregate labour/capital ratio, we can use a similar approach. Defining $\theta_{i,t} = \nu_{i,t}/\kappa_{i,t}$, the aggregate labour capital ratio ($\bar{\theta}_t$) can (implicitly) be written as :

$$\bar{\theta}_t^{\rho_p} = \sum_{i=-\infty}^{t} S_{i,t} \cdot (\theta_{i,t})^{\rho_p} = \sum_{i=-\infty}^{t} S_{i,t} \cdot (\nu_{i,t})^{\rho_p} \cdot \zeta_{i,t} = \sum_{i=-\infty}^{t} \frac{S_{i,t} \cdot (\nu_{i,t})^{\rho_p}}{B_{i,t}^p} - \sum_{i=-\infty}^{t} \frac{A_{i,t}^p}{B_{i,t}^p} \cdot S_{i,t} \tag{36}$$

$$= \xi_t^{\rho_p} \cdot \sum_{i=-\infty}^{t} S_{i,t} \cdot \frac{(A_{i,t}^p)^{\frac{\rho_p}{1+\rho_p}}}{B_{i,t}^p} - \sum_{i=-\infty}^{t} \frac{A_{i,t}^p}{B_{i,t}^p} \cdot S_{i,t} = \xi_t^{\rho_p} \cdot T_{3,t} - T_{4,t}$$

where we have substituted equation (29). Again, the terms $T_{3,t}$ and $T_{4,t}$ can be obtained by means of a recursive update mechanism. Introducing disembodied technical change into (36), we immediately obtain :

$$T_{3,t} = T_{3,t-1} \cdot \frac{(1-\delta) \cdot K_{t-1}}{K_t} \cdot \left(\frac{\left((1+\gamma_n) \cdot (1+\hat{h}_t)^{\varepsilon_n}\right)^{\frac{\rho_p}{1+\rho_p}}}{(1+\gamma_l) \cdot (1+\hat{h}_t)^{\varepsilon_l}} \right)^{-\rho_p} + \frac{A_{t,t}^{p\ \frac{\rho_p}{1+\rho_p}}}{B_{t,t}^p} \cdot \frac{I_{t,t}}{K_t} \tag{37}$$

$$T_{4,t} = T_{4,t-1} \cdot \frac{(1-\delta) \cdot K_{t-1}}{K_t} \cdot \left(\frac{(1+\gamma_n) \cdot (1+\hat{h}_t)^{\varepsilon_n}}{(1+\gamma_l) \cdot (1+\hat{h}_t)^{\varepsilon_l}} \right)^{-\rho_p} + \frac{A_t^p}{B_t^p} \cdot \frac{I_{t,t}}{K_t}$$

Equations (35), (36) and (37) can be used to obtain total capacity labour demand and total production capacity as :

$$N_t = \bar{\theta}_t \cdot K_t = \left\{ \xi_t^{\rho_p} \cdot T_{3,t} - T_{4,t} \right\}^{1/\rho_p} \cdot K_t \tag{38}$$

$$X_t = \pi_t \cdot K_t = \left\{ T_{1,t} - \xi_t^{-\rho_p} \cdot T_{2,t} \right\}^{1/\rho_p} \cdot K_t$$

2.7 Concluding Remarks

Of course, the replacement of a full putty-semi-putty vintage model by its RUM representation has its price. **First** of all, for ex-post elasticities of substitution not equal to 0.5, the RUM model is only an approximation of the full vintage model (although a good one), while **secondly** the terms $T_{1,t}$ through $T_{4,t}$ are recursively defined, and hence need to be initialised. However, in a growing economy it follows that the term $(1-\delta)\cdot K_{t-1}/K_t = (1-\delta)/(1+g)$ (where g is the rate of growth of the capital stock) is smaller than one, and it is easily seen that the influence of any initial value of the individual terms $T_{1,t}$ through $T_{4,t}$ tends to diminish over time. Moreover, this happens more rapidly when either the rate of technical decay or the rate of growth of the economy is high. Nonetheless, for short sample periods, initial values $T_{1,0}$..$T_{4,0}$ will have to be determined next to the other parameters of the production model. We will come back to this later.

The logic of the model is now as follows. First, the technological characteristics of the ex-ante function together with expectations regarding factor prices and technological change, determine the initial technique on the 'newest' ex-post production function. This in turn determines the reference value for marginal labour productivity to be used for the allocation of labour to existing equipment. Thus we obtain the level of output on existing machinery for a given value of the stock of existing capital, as well as the associated amount of labour. Then we obtain the amount of output to be produced on the newest equipment as the difference between the total amount of output required, and the amount of output to be produced on existing equipment. After that, the required amount of investment as well as the associated amount of labour on new equipment can be obtained from the optimum values of the factor productivities on new equipment.

3 The Sectoral Classification, Estimation Procedures, Data Sources and Data Preparations

3.1 Sectoral Classification

The sectoral classification we have used is the same as the one of the EC HERMES models (d'Alcantara and Italianer (1982)). This classification is both detailed enough to take account of 'natural' technological and employment differences between sectors of industry, as well as small enough to ensure that EC-wide modelling efforts based on this classification are feasible in practical / data terms.

The sectoral classification consists of nine sectors of industry, i.e. the agricultural sector (A), the building and construction sector (B), the consumer goods producing sector (C), the energy producing sector (E), the capital goods producing sector (K), the commercial / market services sector (L), the non-market services sector (N), the intermediate goods producing sector (Q) and the communication and transportation sector (Z). The capitals in parentheses are used to denote these sectors in the rest of the paper. A complete account of the composition of the sectors of industry in terms of the Eurostat NACE R25 and R44 classification is provided in appendix B.

3.2 The Estimation Procedure)

3.2.1 Introduction

The RUM model is non-linear in its parameters. Moreover, it is strictly recursive in time, while, in addition, the terms $T_{1,t}$ - $T_{4,t}$ can not be observed directly. Hence, ordinary least squares or related estimation procedures can not directly be used to estimate the parameters of RUM.

Since we have very little a priori information about both the parameters of RUM and the initial values of the terms $T_{1,t}$ - $T_{4,t}$, we have applied a two stage estimation procedure where the first stage is a direct search routine based on a genetic algorithm by Goldberg (1989). Test-runs with both the genetic algorithm and the complex direct search algorithm which is described in Bunday and Garside (1987) and which was used in Meijers and van Zon (1991), showed that the former algorithm outperformed the latter algorithm in almost all cases. The genetic algorithm was optimized and tailored to our needs during a number of test-experiments. The second stage is an ordinary Newton (steepest descent) gradient method.

The application of a direct search method over a relative large parameter space results in a relatively fast convergence to an optimum with global characteristics, whereas the application of a Newton gradient method ensures that local improvements of this optimum, if at all possible, are obtained in a relatively efficient way.

3.2.2 Outline of the Genetic Search Algorithm

The ’body’ of the genetic search algorithm was directly obtained from Goldberg (1989). We represented the RUM parameters as 32 bit bit-strings which were interpreted as a positive fraction of the largest positive integer number which can be represented using 32 bits. Then this fraction was used to obtain the corresponding value of the parameter as follows : $p_i = l_i + f_i. (h_i - l_i)$, where p_i represents the value of parameter i, f_i represents the fraction mentioned above, while l_i and h_i are the low- and high bounds of parameter i, respectively. This proved to be a very efficient and flexible way of representing a specific sub-space of the parameter space in terms of a ’genetic code’.

Using this bit-string representation of the RUM parameters, we changed the reproduction process somewhat in order to increase the chance on reproduction of particularly fit bit-strings. Rather than exchanging a specific bit sub-string of the one parent with a bit sub-string of the other parent, we decided to perform the exchange at the bit level. Moreover, we ensured that the chance on survival of a specific bit was directly and positively related to the fitness of the respective 'supplying' parents.[15] Hence, the bit by bit correspondence between the fittest 'parent-string' and the 'child-string' is (on average) larger than the correspondence with the less fit 'parent-string'.

In addition, we decided to make one copy of the fittest bit-string without altering it in any way, and transferred that copy to the 'new generation'. Thus we ensure that no valuable information is lost due to mating or random mutations. We also let the fittest bit-string mate with other strings from the 'old generation'[16] at least 5 times, where the probability of the selection of a specific partner is proportional to its fitness. The rest of the 'new generation' was created by a random mating process involving two partners from the 'old generation', with selection probabilities proportional to their respective fitness. The resulting bit-string then enters the 'new generation'.

This improved the speed of convergence of the search procedure considerably, while the introduction of a positive mutation rate still ensured enough genetic variation in the gene-pool.

3.2.3 The Objective Function and It's Evaluation

The objective function has been specified in terms of the relative errors between the estimated time-series for production capacity and employment, and their observational counterparts. More in particular, the objective function (F) is equal to :

$$F = \sqrt{\frac{(RMSE_x)^2 + (RMSE_n)^2}{2}} \tag{39}$$

$$RMSE_x = \sqrt{\frac{\sum_{t=t1}^{t2}\left(\frac{x'_t}{x_t} - 1\right)^2}{t2 - t1 + 1}}$$

where a prime associated with a variable denotes its estimated value.

With respect to the generation of initial values for the terms $T_{1,t}$ - $T_{4,t}$ we have used the following strategy. Rather than 'estimating' initial values for the terms $T_{1,t}$-$T_{4,t}$, we have employed equations (35), (37) and (38) directly in order to obtain a linear system in the terms $T_{1,t}$-$T_{4,t}$ which can directly be solved for $T_{1,t}$-$T_{4,t}$ given some initial estimate of the parameter vector for any two pairs of observations on N_t/K_t and X_t/K_t. Note that the recursive adjustment equations with respect to $T_{1,t}$-$T_{4,t}$ can be written as :

$$T_{i,t} = \alpha_{i,t} \cdot T_{i,t-1} + \beta_{i,t} \quad i = 1..4 \tag{40.A}$$

15 A direct measure of the fitness of some parent is provided by the inverse of the function value which is associated with the bit-strings which belong to that parent.

16 A generation always contains 100 individuals.

$$\sigma_t = \left(\frac{X_t}{K_t}\right)^{\rho_p} = T_{1,t} - \gamma_{2,t} \cdot T_{2,t} \tag{40.B}$$

$$\tau_t = \left(\frac{N_t}{K_t}\right)^{\rho_p} = \gamma_{3,t} \cdot T_{3,t} - T_{4,t} \tag{40.C}$$

where the α's , β's and γ's, as well as σ and τ are implicitly defined by the equivalence between (40.A) on the one hand and (35), (37) and (38) on the other. Substitution of (40.A) into (40.B) and (40.C) leads directly to :

$$\begin{bmatrix} T_{1,t-1} \\ T_{2,t-1} \end{bmatrix} = \begin{bmatrix} 1 & -\gamma_{2,t-1} \\ \alpha_{1,t} & -\gamma_{2,t} \cdot \alpha_{2,t} \end{bmatrix}^{-1} \cdot \begin{bmatrix} \sigma_{t-1} \\ \sigma_t - \beta_{1,t} + \gamma_{2,t} \cdot \beta_{2,t} \end{bmatrix} \tag{41.A}$$

$$\begin{bmatrix} T_{3,t-1} \\ T_{4,t-1} \end{bmatrix} = \begin{bmatrix} \gamma_{3,t-1} & -1 \\ \alpha_{3,t} \cdot \gamma_{3,t} & -\alpha_{4,t} \end{bmatrix}^{-1} \cdot \begin{bmatrix} \tau_{t-1} \\ \tau_t - \gamma_{3,t} \cdot \beta_{3,t} + \beta_{4,t} \end{bmatrix} \tag{41.B}$$

Equations (41.A) and (41.B) provide a 'fixed point' for the application of the recursive adjustment equations in order to arrive at a corresponding initial value for the year just before the estimation period. However, since the terms $T_{1,t}$ - $T_{4,t}$ are actually infinite summations over non-negative terms, it follows that the initial values thus obtained can not be negative. Nonetheless, some of them were. In such a situation, we set the initial value equal to zero, and calculated the corresponding 'T-sequence' forward in time again using (40.A). Then the complementary T-term was recalculated using either equation (40.B) or (40.C) and a new initial value (consistent with the imposed zero initial value of the other term) for the associated T-term was obtained. Since any two consecutive observations can be used in this procedure, we took the arithmetical average of all the initial values by T-term as the final initial value to be used in the model estimations / calculations. [17]

3.2.4 Labour Hoarding

RUM generates time-series regarding capacity labour demand. Unfortunately, observations on capacity labour demand are not available in practice. The only time-series which are available are those concerned with employment. Hence, in order to be able to estimate the parameters of RUM, it is necessary to hypothesize some relation between

17 Note that a change in the parameter-vector to be estimated, also implies a corresponding change in the initial values of the T-terms. Given these T-sequences and the parameter-vector as well as the investment shares, it is now straightforward to obtain estimates of production capacity and capacity labour demand.

capacity labour demand and employment. Following Muysken and van Zon (1987) to some extent [18], we assume that labour hoarding may occur in times of under-utilisation of production capacity. More in particular, we assume that actual employment is equal to minimum employment (as given by the product of the rate of capacity utilisation and capacity labour demand) plus a positive fraction of the difference between capacity labour demand and minimum labour demand, i.e. :

$$E_t = q_t \cdot N_t + \chi \cdot (N_t - q_t \cdot N_t) = N_t \cdot (q_t + \chi \cdot (1 - q_t)) \tag{42}$$

where E_t is employment, N_t is capacity labour demand and q_t is the rate of capacity utilisation. χ is the hoarding parameter, which is assumed to be constant over time. Equation (42) is also needed to link the unobserved ratio N_t / K_t with the observed ratio E_t / K_t. Hence, (42) also enables us to calculate the terms $T_{1,t}$-$T_{4,t}$.

3.3 Data Sources and Preliminary Data Manipulations

The data on employment, wage-cost per worker, valued added at factor cost, both in current and in constant prices were obtained from the SEC2 domain from the CRONOS database provided by Eurostat.[19] This also goes for investment by ownership branch in constant and in current prices, except for the Netherlands, where the latter variables were obtained from the Central Planning Bureau. Data on working hours by sector of industry were obtained from the SOCI domain of the CRONOS database in the case of Germany and from the Central Planning Bureau for the Netherlands. Finally, data on capital depreciation charges by sector of industry in current prices were obtained from the input-output tables for both countries. Their CRONOS equivalents were obtained by multiplying the share of depreciation charges in the input-output value added at current market prices with the observations of value added at current market prices in the CRONOS database. The data on the nominal long term rate of interest were obtained from IMF (1991). We have taken the yield on long term government bonds as a direct indicator of the long term rate of interest.

Unfortunately, observations on rates of capacity utilisation are not available within the CRONOS database. Hence, these time-series had to be constructed. More in particular, we have assumed that the three year moving average of the rate of growth of real value

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18 Muysken and van Zon distinguish between two types of hoarding, i.e. forced and desired (ex-post). For reasons of simplicity we only use desired hoarding.

19 CRONOS is a ’harmonised’ database covering all individual member countries of the EC. Hence, given our intention to expand our vintage modelling exercises to the rest of the EC, we have chosen the CRONOS database as our central data-source. Unfortunately, the harmonisation of the data has reduced the level of sectoral detail which is available in practice. More in particular, the HERMES classification we have adhered to, is about the most detailed classification one can come up with within the context of CRONOS.

added at factor costs can be used as an indicator of the rate of growth of capacity output. The rate of growth of the rate of capacity utilisation is by definition equal to the difference between the rate of growth of real output (as measured by real value added at factor cost) and real capacity output. Hence, we can obtain an estimate of the rate of capacity utilisation by renormalising a first round estimate of the rate of capacity utilisation based on the repeated application of its 'estimated' rate of growth to some initial value, such that its maximum 'observed' value becomes equal to one. This approach is comparable in nature to the Bischoff-approach (c.f. Bischoff (1971), d'Alcantara and Italianer (1982)).

Unfortunately, observations on the sectoral capital stocks are not available. However, assuming that capital depreciation charges are valued at replacement costs, while at the same time the capital stocks themselves evolve in accordance with the perpetual inventory method, we have been able to construct capital stock time-series in the following way: [20]

$$K_t = (1-\delta)\cdot K_{t-1} + I_t = K_o \cdot (1-\delta)^t + \sum_{i=1}^{t} (1-\delta)^{t-i} \cdot I_i \tag{43}$$

Assuming depreciation charges in current prices to be given by D_t, while the price index of investment is given by P_t, it follows immediately that depreciation charges valued at replacement costs are equal to :

$$D_t = \delta \cdot K_{t-1} \cdot P_t \;\Rightarrow\; K_t = \frac{D_{t+1}}{P_{t+1}} \cdot \frac{1}{\delta} \tag{44}$$

Substitution of (44) into (43) yields :

$$D_t - \frac{P_t}{P_{t-1}} \cdot D_{t-1} = \delta \cdot \frac{P_t}{P_{t-1}} \cdot (P_{t-1} \cdot I_{t-1} - D_{t-1}) \tag{45}$$

Equation (45) can be used to estimate δ by means of ordinary least squares. The estimation results for the Netherlands and Germany are given in table 1 below.

As can be seen from the table above, the estimation results are quite similar for both countries. The highest rate of depreciation is found for the Building and Construction sector in both cases, while low values are found for the L- and N-sector. Intermediate values are found for the industrial sectors and the Z-sector.[21]

20 See also van Zon (1986).

21 In case of the Netherlands we might have hit upon a 'rule of thumb' used by the Central Bureau of Statistics in determining depreciation charges by the Z-sector.

Table 1. Technical decay parameters for Germany and the Netherlands

	Germany			the Netherlands		
Sector	δ	t-value	R^2 Adj.	δ	t-value	R^2 Adj.
A	0.046889	2.35	0.996	0.040092	9.90	0.999
B	0.138229	3.23	0.875	0.114175	4.44	0.998
C	0.028584	2.03	0.987	0.052072	9.30	0.999
E	0.045942	5.01	0.993	0.034292	1.72	0.999
K	0.097377	9.39	0.996	0.052389	13.37	0.999
L	0.048133	13.95	0.998	0.016752	11.44	0.999
N	0.029698	7.79	0.992	0.009902	6.94	0.998
Q	0.076698	1.62	0.885	0.071354	4.44	0.991
Z	0.053974	9.63	0.997	0.100000	6.31	0.997
Sample	1978-1988			1970-1990		

These estimated values of the decay parameters were then used to obtain initial values for the sectoral capital stocks in the following way. From equations (43) and (44) it follows that :

$$K_0(t) = \frac{\frac{D_{t+1}}{P_{t+1}\cdot\delta} - \sum_{i=1}^{t}(1-\delta)^{t-i}\cdot I_i}{(1-\delta)^t} \tag{46}$$

Since the RHS of (46) consists of observable variables next to the estimated value of the depreciation parameter δ, it follows that for any point in time>0 a corresponding value of $K_0(t)$ can be obtained. We took the arithmetical average over all different K0(t)'s as the initial value of the sectoral capital stock time-series to be constructed and then generated the data in accordance with (43).

4 Estimation Results

4.1 Results for Germany

The estimation results for Germany are listed in tables 2-10 below. These tables contain five columns. The first column holds the names of the parameters which were estimated. The second column contains the parameter vector which minimizes the objective function. The third column contains the average values of the parameters as they were obtained during 10 consecutive estimation rounds using the genetic search algorithm and the Newton steepest descent method in a consecutive fashion, each time starting from a different random population of initial parameter vectors. The fourth column contains the value of the standard-deviation by parameter obtained from the aforementioned 10 experiments, while the fifth column contains the ratio of the average value of the parameter and its corresponding standard deviation. It is to be noted that the interpretation of the standard deviation in this particular estimation set-up is different from the standard interpretation, i.e. it does not refer to the standard deviation associated with the 'best' parameter vector, but it refers to the entire sample of estimated parameter vectors. Still, the calculated standard deviation can be used as an indicator of the robustness of a particular estimate, since a small value of the standard deviation does point to a relatively strong tendency of the parameter in question to be located in a particular sub-space of the total parameter space.

Apart from these 'quasi-statistics', we also present the values of the root mean squared errors associated with the 'best' parameter vector. Moreover, we have included the ratio of the elasticity of substitution ex-post and the elasticity of substitution ex-ante as a direct indicator of the remaining scope for substitution after the moment of installation of new equipment.

In order to avoid the occurrence of computational problems, we decided to put some lower- and upper limits on the parameter values to be determined. More in particular, we have put an upper-limit of 6 percent and a lower-limit of 0 percent on the different rates of technical change. The ρ's ex-ante and ex-post are in between 0.3 and 5, which implies a range of variation for the elasticities of substitution ex-ante and ex-post of about 0.17-0.77. A_0 is in between 0 and 10, while B_0 is in between 0.01 and 100. The hoarding parameter χ is assumed positive and at most equal to 1. The risk-premium τ is positive and at most equal to 10 percent. [22]

The results of the A-sector indicate first of all that the fit is reasonable, while the estimates themselves are relatively robust, with the exception of the rate of embodied capital augmenting technical change μ_I , as well as the CES distribution parameters A_0 and B_0. As regards the standard deviations of χ and τ, it is clear that these are relatively small due to the clustering of the estimates at their respective upper-limits. Note too, that the hoarding parameter in the A-sector is almost equal to 1 which is significantly higher than in the other sectors. This is presumably a consequence of the fact that the agricultural sector has a high employment share of self-employed people (in 1990, 78

22 Note that the fact that we have imposed boundaries on the parameter space to be searched may lead to a clustering of estimates at a specific parameter boundary. In such a case the calculated sample standard deviation may be relatively low, and needs to be interpreted with extra caution.

Table 2. Estimation Results for Sector A

Param	Best	Avg.	St.D.	Avg. / St.D.
γ_I	0.028569	0.029976	0.007346	4.080869
γ_n	0.053387	0.059068	0.002050	28.814329
μ_I	0.046568	0.033978	0.020377	1.667460
μ_n	0.045208	0.046350	0.015344	3.020790
σ_a	0.405616	0.479983	0.106068	4.525225
σ_p	0.267277	0.286747	0.039307	7.295001
A_0	0.000420	0.020605	0.033675	0.611872
B_0	12.854958	42.991704	29.911187	1.437312
χ	0.963664	0.939311	0.016518	56.864663
τ	0.099425	0.097443	0.002821	34.541590
F	0.034716	0.041141	0.003908	10.527526

$RMSE_x = 0.02516$, $RMSE_n = 0.04216$, $\sigma_p/\sigma_a = 0.66$

percent of total employment consisted of self-employed people), who would probably be more than willing to 'hoard themselves' in times of under-utilisation. Note that on average the estimated rates of disembodied technical change are only slightly lower than the estimates regarding embodied technical change, whereas the drop in substitution possibilities after the moment of installation of new investment is considerable.

Results similar to the A-sector are obtained for the B-sector, although the nature of technical change seems to be slightly more capital augmenting than labour augmenting. The hoarding parameter χ is much lower, though.

Table 3. Estimation Results for Sector B

Param	Best	Avg.	St.D.	Avg. / St.D.
γ_I	0.049990	0.053622	0.004601	11.655486
γ_n	0.039145	0.031412	0.009292	3.380736
μ_I	0.059238	0.051068	0.012881	3.964580
μ_n	0.033981	0.021786	0.011310	1.926239
σ_a	0.441527	0.410923	0.048493	8.473941
σ_p	0.248223	0.295782	0.027584	10.722897
A_0	2.856702	1.340818	1.825022	0.734686
B_0	64.845892	45.612367	37.871371	1.204402
χ	0.468623	0.430222	0.223928	1.921250
τ	0.083419	0.056554	0.028259	2.001300
F	0.021136	0.025812	0.003104	8.316841

$RMSE_x$ = 0.01849, $RMSE_n$ = 0.02349, σ_p/σ_a = 0.56

The E-sector is totally different from the other sectors of industry. First of all, embodied technical change is negligible in comparison with the other sectors, but also in comparison with disembodied technical change in the E-sector itself. Again, substitution possibilities ex-post are far smaller than those ex-ante, while at the same time the hoarding parameter is relatively high. The latter may reflect the fact that the seasonal variation in the demand for energy requires some built-in slack in the capacity to supply. Note too that B_0 has reached its upper-limit.[23]

23 This is a fairly common phenomenon, also for the other sectors of industry, as one will see shortly. During our estimation exercises it appeared that some of the information regarding the scale of (capacity) output and employment is already supplied by our procedure to calculate the initial values of $T_{1,0}$-$T_{4,0}$. More in particular, we noticed that once we started using the 'T-term initialisation procedure' as outlined in section 3.2.3, the Newton method showed a tendency to blow up the A_0 and B_0 parameters in exactly equal proportions, without, however, decreasing the objective function in any significant way. At the same time, we ended up with numerical anomalies due to this explosive process. Hence, the computational need arose to impose upper-limits on A_0 and B_0. Fortunately, the practical consequences of this decision in terms of changes in the value of the objective function were non-existent.

Table 4. Estimation Results for Sector E

Param	Best	Avg.	St.D.	Avg. / St.D.
γ_I	0.002085	0.011069	0.013296	0.832516
γ_n	0.045235	0.033008	0.008920	3.700648
μ_I	0.000355	0.003217	0.003524	0.912843
μ_n	0.005852	0.005476	0.005196	1.053773
σ_a	0.462892	0.510763	0.107811	4.737574
σ_p	0.272951	0.316136	0.074958	4.217503
A_0	0.015708	0.029056	0.033829	0.858906
B_0	100.000000	77.858440	36.880424	2.111105
χ	0.823801	0.864267	0.073387	11.776807
τ	0.093788	0.075539	0.023135	3.265199
F	0.039737	0.045031	0.003275	13.748707

$RMSE_x = 0.03870$, $RMSE_n = 0.04074$, $\sigma_p/\sigma_a = 0.59$

In figures 2 and 3 below, we have drawn the estimated and the actual time-series for employment and production capacity as they are associated with the parameter-estimates presented above. In these figures the RUM-estimates have the post-fix ’CALC’. The corresponding data do not have a such post-fix. The sectors to which the individual time-series correspond are indicated by the last character of the names of the time-series in question. Looking at the figures, we conclude that the Building and Construction sector performs the best both with respect to employment and production capacity, whereas the other sectors still perform in a reasonable way at least over the medium and longer run.

The results of the industrial sectors C, K and Q are presented below.

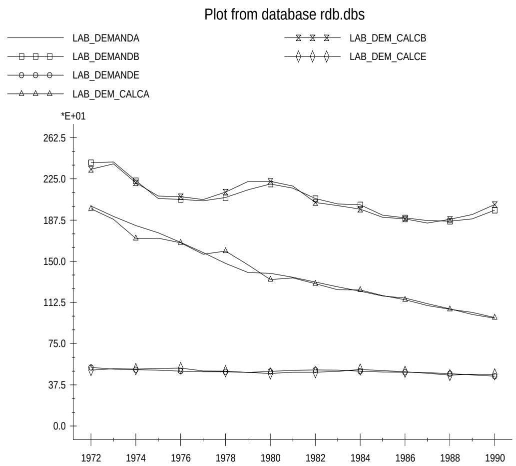


Fig. 2 Employment in the A, B and E sectors

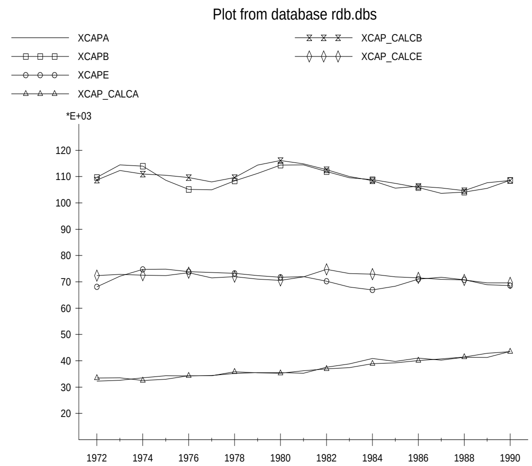


Fig. 3 Production capacity in the A, B and E sectors

Table 5. Estimation Results for Sector C

Param	Best	Avg.	St.D.	Avg. / St.D.
γ_I	0.007667	0.021240	0.011738	1.809499
γ_n	0.021627	0.035899	0.013970	2.569736
μ_I	0.026034	0.029005	0.010935	2.652409
μ_n	0.003529	0.027258	0.016587	1.643375
σ_a	0.460520	0.530654	0.063262	8.388161
σ_p	0.245653	0.391663	0.119747	3.270759
A_0	0.081245	0.651671	0.717311	0.908491
B_0	99.999316	80.570881	27.368382	2.943940
χ	0.291261	0.226188	0.072821	3.106080
τ	0.036319	0.039022	0.024454	1.595723
F	0.036020	0.037805	0.001749	21.619009

$RMSE_x = 0.03000$, $RMSE_n = 0.04117$, $\sigma_p/\sigma_a = 0.53$

For the C-sector the nature of embodied technical change is clearly different from the nature of disembodied technical change, although the average values of the parameter estimates have similar magnitudes : embodied technical change is mainly capital augmenting, while disembodied technical change is mainly labour augmenting. Again, the drop in substitution possibilities is considerable, while the hoarding parameter is relatively low.

Table 6. Estimation Results for Sector K

Param	Best	Avg.	St.D.	Avg./St.D.
γ_I	0.030173	0.030516	0.001789	17.061952
γ_n	0.029815	0.028034	0.002289	12.249131
μ_I	0.060000	0.019782	0.015620	1.266494
μ_n	0.060000	0.027058	0.011996	2.255643
σ_a	0.283055	0.413279	0.046232	8.939271
σ_p	0.196352	0.231049	0.013285	17.392228
A_0	0.004345	0.131225	0.103862	1.263452
B_0	24.553642	80.850330	20.954159	3.858438
χ	0.424407	0.631530	0.111510	5.663447
τ	0.100000	0.048529	0.026831	1.808677
F	0.027957	0.034415	0.002707	12.712224

$RMSE_x$ = 0.02845, $RMSE_n$ = 0.02745, σ_p/σ_a = 0.69

In the K-sector, embodied technical change is very important indeed : in both cases the upper-bounds are active! The rates of disembodied technical change have only half the value of the rates of embodied technical change. Note that the best parameter-vector seems to be somewhat of an outlier as far as embodied technical change is concerned, since the average values of the embodied technical change parameters are far smaller than the 6 percent we have obtained. Note too that nonetheless the objective function value of the best parameter vector is about 20 percent lower than the average value of the objective function, and lies at a distance of about 30 standard-deviations from the sample average. Note in addition, that substitution possibilities ex-ante and ex-post are far lower than in the other sectors of industry. At the same time it should be noted that the estimate of the elasticity of substitution ex-ante is relatively far below its sample average.

The Q-sector is not that different from the K-sector with respect to the estimated values of the elasticities of substitution ex-ante and ex-post. In the Q-sector disembodied technical change is slightly more important than embodied technical change. Moreover

Table 7. Estimation Results for Sector Q

Param	Best	Avg.	St.D.	Avg. / St.D.
γ_I	0.046292	0.052862	0.002979	17.742647
γ_n	0.032257	0.030946	0.002473	12.511529
μ_I	0.034344	0.028415	0.004907	5.790608
μ_n	0.037774	0.034964	0.013270	2.634832
σ_a	0.243091	0.407270	0.064792	6.285771
σ_p	0.191563	0.247161	0.020984	11.778343
A_0	0.000000	0.000706	0.000764	0.924295
B_0	100.000000	87.578556	25.671256	3.411542
χ	0.869999	0.806984	0.071968	11.213059
τ	0.000000	0.032715	0.023018	1.421257
F	0.041746	0.044021	0.001564	28.154846

$RMSE_x = 0.05452$, $RMSE_n = 0.02265$, $\sigma_p/\sigma_a = 0.79$

there seems to be a slight bias in favour of capital augmentation. Note too that the average value of the risk premium is very low indeed, while the best parameter vector even shows a zero risk premium, which is a somewhat unlikely result.

The estimation results are summarised in figures 4 and 5.

Looking at figures 4 and 5, we notice that the K-sector does indeed outperform the C-and Q-sector, but only very slightly. While short term fluctuations are captured far less well than in case of, for instance, the B-sector [24], the performance in the medium and longer run is quite acceptable.

The estimation results for the services sectors L, N and Z are presented in tables 8-10.

24 This is especially apparent from the auto-correlation of errors with respect to production capacity.

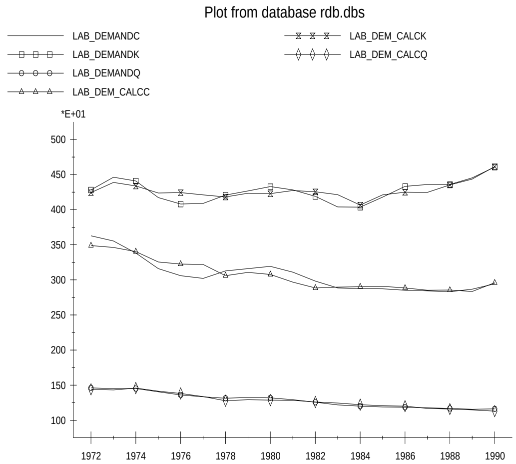


Fig. 4 Employment in the C, K and Q sectors

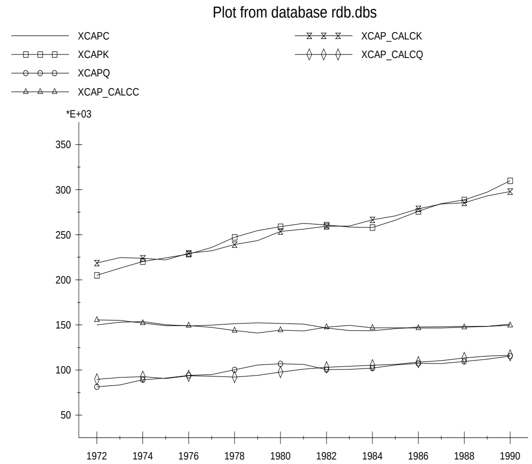


Fig. 5 Production capacity in the C, K and Q sectors

Table 8. Estimation Results for Sector L

Param	Best	Avg.	St.D.	Avg. / St.D.
γ_I	0.028332	0.028096	0.001948	14.424570
γ_n	0.027886	0.029260	0.001319	22.182209
μ_I	0.034905	0.020306	0.007956	2.552266
μ_n	0.008061	0.011689	0.006896	1.695113
σ_a	0.458834	0.465564	0.040235	11.571005
σ_p	0.240415	0.250698	0.005480	45.745643
A_0	0.017205	0.005417	0.005043	1.074265
B_0	97.767632	38.299850	36.837516	1.039697
χ	0.002345	0.041560	0.037099	1.120259
τ	0.082411	0.089974	0.020777	4.330513
F	0.025457	0.028152	0.001249	22.545462

$RMSE_x = 0.02896$, $RMSE_n = 0.02138$, $\sigma_p/\sigma_a = 0.52$

The L-sector is the largest sector of the German economy in terms of its labour share (L accounted for 25 % of total employment in 1970 and 33% in 1990). A striking feature of the results is that the labour hoarding parameter is nearly equal to zero. This also goes for the sample average with respect to labour hoarding. Note too, that the employment effects of the implied 'hiring and firing' flexibility is compensated more or less by the near absence of embodied labour augmenting technical change. Since disembodied technical change is more or less Hicks-neutral, this implies that the overall impact of technical change is relatively capital augmenting.

Table 9. Estimation Results for Sector N

Param	Best	Avg.	St.D.	Avg. / St.D.
γ_I	0.012491	0.015909	0.013236	1.202003
γ_n	0.026644	0.024417	0.006385	3.824112
μ_I	0.040377	0.026870	0.013956	1.925297
μ_n	0.002041	0.004284	0.007863	0.544792
σ_a	0.493512	0.505435	0.059393	8.510073
σ_p	0.348884	0.350618	0.060810	5.765820
A_0	0.020535	0.183222	0.370991	0.493873
B_0	5.075855	31.963648	29.719934	1.075495
χ	0.012349	0.043143	0.074364	0.580164
τ	0.100000	0.094325	0.016275	5.795797
F	0.015640	0.021571	0.003819	5.648496

$\mathrm{RMSE}_x = 0.01683$, $\mathrm{RMSE}_n = 0.01436$, $\sigma_p/\sigma_a = 0.71$

Results comparable in nature to the L-sector are obtained for the N-sector. Again, labour augmenting technical change is virtually non-existent, while embodied capital augmenting technical change is relatively strong. However, in case of the N-sector there is a clear bias in disembodied technical change in favour of labour augmentation. Moreover, the sample standard deviation of capital augmenting disembodied technical change is relatively high. It should be noted that, as in the L-sector, the labour hoarding parameter is nearly equal to zero. Note too, that the N-sector is the best of all sectors taken together in terms of the value of the objective function.

The Z-sector differs from the other services sectors to the extent that there is somewhat more embodied labour augmenting technical change. Disembodied technical change is of the same magnitude as in the L- and N-sector. Substitution possibilities ex-post and ex-ante are very similar to those in the L-sector and somewhat smaller than those

Table 10. Estimation Results for Sector Z

Param	Best	Avg.	St.D.	Avg. / St.D.
γ_I	0.026631	0.028864	0.000943	30.620095
γ_n	0.029356	0.027204	0.001993	13.649918
μ_I	0.024935	0.014950	0.007005	2.134282
μ_n	0.014416	0.023349	0.011085	2.106433
σ_a	0.406138	0.410124	0.019093	21.480502
σ_p	0.266859	0.262808	0.014935	17.596298
A_0	0.000409	0.000259	0.000139	1.855905
B_0	6.674865	8.084044	1.407952	5.741702
χ	0.307042	0.337204	0.093763	3.596336
τ	0.045772	0.049441	0.016818	2.939807
F	0.031469	0.031914	0.000301	106.137330

$RMSE_x = 0.03278$, $RMSE_n = 0.03010$, $\sigma_p/\sigma_a = 0.66$

in the N-sector. The main difference between the Z-sector and the other services sectors, however, is that the hoarding parameter is definitely larger than zero, although less (on average) than in the non-services sectors.

Figures 6 and 7 below show the correspondence between estimated time-series and the associated data.

Again, it is clear that medium and longer term developments are captured in a reasonable way, especially with respect to employment. The estimated value of production capacity in the L-sector seems to show some counter-cyclical movements, while in addition estimation errors have a tendency to persist for longer periods of time.

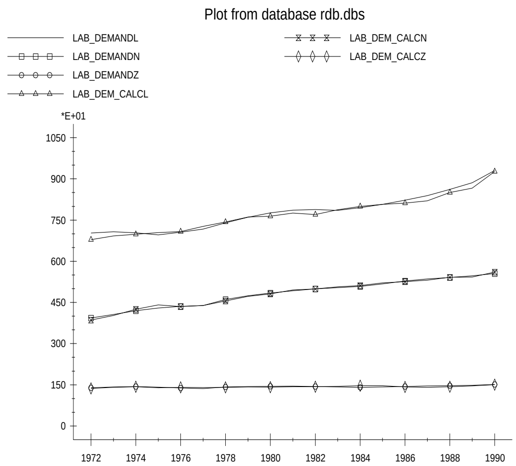


Fig. 6 Employment in the L, N and Z sectors

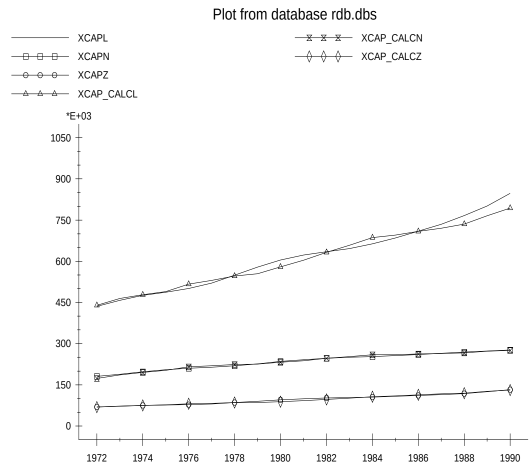


Fig. 7 Production capacity in the L, N and Z sectors

4.2 Results for the Netherlands

The estimation results for the Netherlands are listed in tables 11-19 below.

Table 11. Estimation Results for Sector A

Param	Best	Avg.	St.D.	Avg. / St.D.
γ_I	0.028505	0.031400	0.005368	5.849253
γ_n	0.058735	0.059726	0.000508	117.506676
μ_I	0.031169	0.041709	0.011538	3.614940
μ_n	0.052291	0.046238	0.010922	4.233502
σ_a	0.282436	0.416236	0.083394	4.991193
σ_p	0.270346	0.273134	0.039537	6.908398
A_0	0.000015	0.011858	0.019475	0.608899
B_0	100.000000	72.849441	32.461051	2.244211
χ	0.991489	0.964687	0.035731	26.998556
τ	0.094595	0.093840	0.007408	12.666562
F	0.043863	0.046871	0.002090	22.425242

$RMSE_x$ = 0.04855, $RMSE_n$ = 0.03865, σ_p/σ_a = 0.96

With respect to the A-sector, the Netherlands show results which are quite similar to those of Germany, although the elasticity of substitution ex-ante is far smaller in case of the Netherlands. The rates of disembodied technical change are almost identical, whereas there is only a relatively small difference between the rates of embodied technical change : in Germany only the rate of embodied capital augmenting technical change is somewhat larger than in the Netherlands. Note, moreover, that in both cases the hoarding parameter is nearly equal to 1.

Table 12. Estimation Results for Sector B

Param	Best	Avg.	St.D.	Avg. / St.D.
γ_I	0.052186	0.042980	0.014405	2.983793
γ_n	0.042907	0.027244	0.007728	3.525422
μ_I	0.008568	0.024525	0.015941	1.538491
μ_n	0.047055	0.025214	0.013527	1.863932
σ_a	0.488756	0.448602	0.045050	9.957939
σ_p	0.412083	0.342753	0.060447	5.670309
A_0	0.015204	0.172340	0.302332	0.570037
B_0	0.246238	20.186446	37.358723	0.540341
χ	0.781181	0.651235	0.238547	2.730005
τ	0.087976	0.032261	0.037333	0.864139
F	0.029274	0.036394	0.004662	7.806857

$RMSE_x = 0.02693$, $RMSE_n = 0.03145$, $\sigma_p/\sigma_a = 0.84$

In the B-sector the main difference between Germany and the Netherlands lies in the nature of embodied technical change : it is mainly labour augmenting in the Netherlands and primarily capital augmenting in Germany. Moreover, in the Netherlands the elasticity of substitution does not drop as far as is the case in Germany, while in addition the hoarding parameter is significantly higher than in Germany.

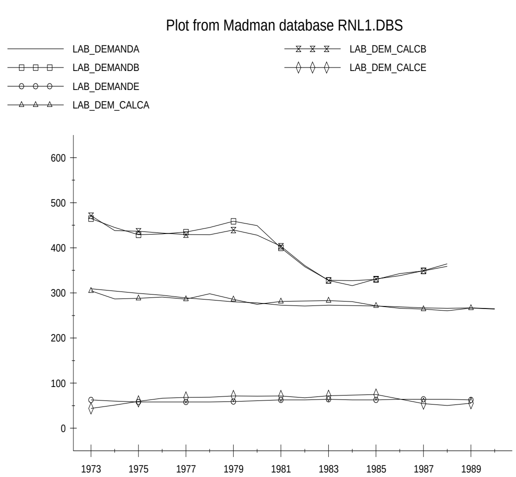
In case of the Netherlands, no satisfactory results could be obtained for the E-sector. This becomes apparent when one looks at the value of the objective function which differs by an order of magnitude from the other sectors. Hence we will not discuss these results any further, save for the fact that also in the Netherlands embodied technical change is apparently of negligible importance here.

Table 13. Estimation Results for Sector E

Param	Best	Avg.	St.D.	Avg. / St.D.
γ_I	0.044996	0.047668	0.008331	5.721429
γ_n	0.046957	0.025817	0.018880	1.367404
μ_I	0.000634	0.034403	0.026768	1.285208
μ_n	0.000098	0.024995	0.018384	1.359598
σ_a	0.591908	0.661394	0.061641	10.729781
σ_p	0.509277	0.520330	0.090146	5.772098
A_0	0.148082	0.233782	0.199673	1.170823
B_0	100.000000	70.846197	41.840022	1.693264
χ	0.034946	0.149496	0.149380	1.000773
τ	0.099830	0.098848	0.000866	114.081493
F	0.219673	0.295124	0.058804	5.018785

$RMSE_x = 0.20483$, $RMSE_n = 0.23357$, $\sigma_p/\sigma_a = 0.86$

The fit of the RUM-model in the case of the A, B and E sectors is summarised in figures 8 and 9 below.



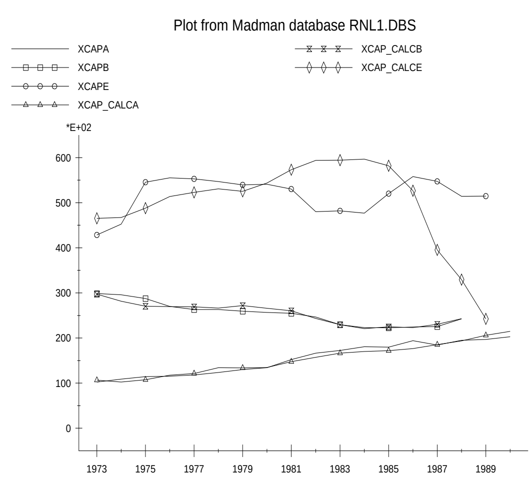


Fig. 8 Employment in the A, B and E sectors

Fig. 9 Production capacity in the A, B and E sectors

Again, we notice that with respect to the generation of employment, RUM performs very well in case of the B-sector. However, it should be recalled here that the B-sector shows the largest rate of technical decay both in Germany as well as in the Netherlands. Hence, the influence of the 'old' capital stock is felt less severely in case of the B-sector than in case of the other sectors. The latter may explain the relative lack of the ability of the L- and N-sector to follow short term cyclical behaviour, despite the fact that, also in case of the Netherlands, the hoarding parameters are relatively low in comparison with the other sectors (except for E) but still much higher than in Germany.

The estimation results for the C-, K- and Q-sectors are presented in tables 14-16.

Table 14. Estimation Results for Sector C

Param	Best	Avg.	St.D.	Avg. / St.D.
γ_I	0.019371	0.028360	0.012459	2.276282
γ_n	0.056281	0.047320	0.012982	3.645195
μ_I	0.055848	0.053344	0.007513	7.099794
μ_n	0.031431	0.034317	0.014104	2.433118
σ_a	0.493779	0.520699	0.096171	5.414321
σ_p	0.275571	0.319740	0.064271	4.974877
A_0	0.212441	0.569923	0.697155	0.817498
B_0	100.000000	94.459342	16.595259	5.691948
χ	0.665827	0.736948	0.154753	4.762082
τ	0.082223	0.085534	0.020232	4.227621
F	0.024037	0.030852	0.003925	7.860764

$RMSE_x = 0.02871$, $RMSE_n = 0.01820$, $\sigma_p/\sigma_a = 0.56$

Technical change in the C-sector is far more important in the Netherlands than in Germany : although the biases have the same sign in both cases, the average levels in the Netherlands are about 3 percentage points higher than in Germany. Substitution characteristics are very similar in the Netherlands and Germany, though.

Note that with respect to technical change in the K-sector, the results for the Netherlands are almost the same as in Germany : embodied technical change is (nearly) at its upper-bound, whereas disembodied technical change is (about) half the level of

Table 15. Estimation Results for Sector K

Param	Best	Avg.	St.D.	Avg./St.D.
γ_I	0.027277	0.035408	0.015466	2.289463
γ_n	0.019337	0.032614	0.013075	2.494394
μ_I	0.059014	0.054673	0.005648	9.679634
μ_n	0.059956	0.051585	0.013647	3.779905
σ_a	0.322330	0.477091	0.116723	4.087372
σ_p	0.263951	0.308786	0.076369	4.043319
A_0	0.010177	0.576874	0.591158	0.975836
B_0	100.000000	83.588049	26.266846	3.182264
χ	0.582204	0.684903	0.202425	3.383489
τ	0.096818	0.091317	0.014848	6.150211
F	0.030914	0.035319	0.003714	9.510616

$RMSE_x = 0.02663$, $RMSE_n = 0.03467$, $\sigma_p/\sigma_a = 0.82$

embodied technical change. Note too, that the value of the ex-ante elasticity of substitution is low when compared to the other sectors of industry, but of similar magnitude when compared to Germany.

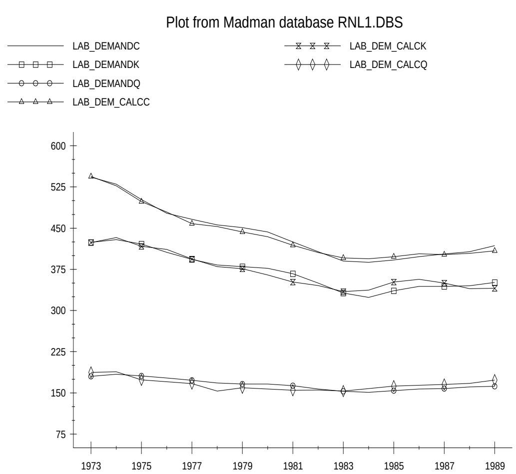
The results for the Q-sector in the Netherlands are totally different from the ones obtained for Germany : in the Netherlands embodied technical change is far less important than disembodied technical change, while this is not the case in Germany. Here disembodied technical change is on average somewhat stronger than embodied technical change, but primarily of the capital augmenting kind, whereas in case of the Netherlands disembodied technical change is strongly labour augmenting. Moreover, in the Netherlands the elasticities of substitution are higher than in the case of Germany. These differences seem to point to a difference in the composition of the Q-sector which is dominated by relatively capital intensive chemical industries in Germany.

The fit of the RUM-model for the C-, K- and Q-sectors is depicted in figures 10 and 11.

Table 16. Estimation Results for Sector Q

Param	Best	Avg.	St.D.	Avg. / St.D.
γ_I	0.038539	0.049524	0.011525	4.297014
γ_n	0.060000	0.054726	0.008929	6.128866
μ_I	0.002457	0.039245	0.020995	1.869256
μ_n	0.005288	0.046225	0.021021	2.198995
σ_a	0.395471	0.474740	0.054423	8.723071
σ_p	0.237031	0.254728	0.013151	19.369967
A_0	0.003059	0.044530	0.080243	0.554938
B_0	90.492962	64.864320	38.156731	1.699944
χ	0.906587	0.916484	0.141539	6.475157
τ	0.100000	0.086185	0.026829	3.212372
F	0.052440	0.062981	0.012537	5.023601

$RMSE_x = 0.03464$, $RMSE_n = 0.06557$, $\sigma_p/\sigma_a = 0.60$



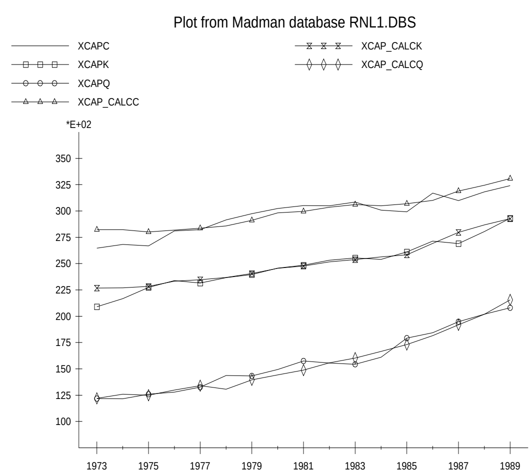


Fig. 10 Employment in the C, K and Q sectors

Fig. 11 Production capacity in the C, K and Q sectors

As was apparent from the values of the root mean squared error terms, the fit of employment is somewhat better than the fit of production capacity. We conclude that medium and longer term movements are captured in a satisfactory way.

The estimation results for the services sectors L, N and Z are presented in tables 17-19.

Table 17. Estimation Results for Sector L

Param	Best	Avg.	St.D.	Avg. / St.D.
γ_I	0.015018	0.020680	0.007458	2.772781
γ_n	0.021036	0.025706	0.007923	3.244298
μ_I	0.059488	0.053660	0.005938	9.036540
μ_n	0.008881	0.024964	0.013751	1.815367
σ_a	0.541554	0.584930	0.064668	9.045123
σ_p	0.277359	0.320048	0.051998	6.154989
A_0	0.162537	0.548952	0.549427	0.999135
B_0	99.818099	98.477232	3.271325	30.103162
χ	0.581314	0.473349	0.112075	4.223497
τ	0.096303	0.086061	0.010496	8.199107
F	0.033055	0.036142	0.002742	13.181638

$RMSE_x = 0.03832$, $RMSE_n = 0.02677$, $\sigma_p/\sigma_a = 0.51$

For the L-sector we observe with respect to technical change that the results are similar to those for Germany : embodied technical change is more important than disembodied technical change and mainly of the capital augmenting kind, while there is hardly any difference between the rates of capital augmenting and labour augmenting disembodied technical change both for Germany and the Netherlands, although disembodied technical change is slightly less important for the Netherlands than for Germany. Substitution characteristics are again quite similar. Only the labour hoarding parameter is far larger in the Netherlands than in Germany.

The results for the N-sector differ from those obtained for Germany in as far as technical change is concerned : only labour augmenting embodied technical change is important in case of the Netherlands, while embodied capital augmenting technical change is important in case of Germany. Moreover, in Germany disembodied technical change matters, whereas this is hardly the case in the Netherlands. Substitution characteristics are again remarkably similar, also in comparison with the other sectors of industry : they are relatively high both in Germany and the Netherlands, while the differences between the ex-ante and ex-post values are very similar indeed.

Table 18. Estimation Results for Sector N

Param	Best	Avg.	St.D.	Avg./St.D.
γ_I	0.000206	0.007165	0.007978	0.898124
γ_n	0.009209	0.017111	0.007707	2.220119
μ_I	0.002106	0.021636	0.016888	1.281154
μ_n	0.045337	0.030847	0.016684	1.848863
σ_a	0.540941	0.539020	0.110970	4.857338
σ_p	0.332262	0.364491	0.069827	5.219943
A_0	0.026877	0.172222	0.311683	0.552555
B_0	92.943156	85.679814	25.870214	3.311910
χ	0.176306	0.388840	0.312057	1.246055
τ	0.100000	0.085165	0.028470	2.991341
F	0.017539	0.022522	0.005120	4.399142

$\mathrm{RMSE}_x = 0.01502$, $\mathrm{RMSE}_n = 0.01974$, $\sigma_p/\sigma_a = 0.61$

For the Z-sector we observe that biases in technical change have the same sign both in Germany and the Netherlands, while the average level of technical change in the Netherlands is about 2 percentage points higher both for embodied and disembodied technical change. Substitution characteristics coincide more or less.

The fit of the RUM-model for the services sectors is depicted in figures 12 and 13.

Table 19. Estimation Results for Sector Z

Param	Best	Avg.	St.D.	Avg./St.D.
γ_I	0.058327	0.058480	0.002846	20.544845
γ_n	0.044473	0.038607	0.008442	4.573296
μ_I	0.045101	0.033884	0.013314	2.545063
μ_n	0.029592	0.026911	0.019195	1.401982
σ_a	0.418927	0.511247	0.079489	6.431691
σ_p	0.252615	0.265035	0.012801	20.703970
A_0	0.016296	0.187631	0.499806	0.375407
B_0	82.827698	33.684758	37.720508	0.893009
χ	0.736967	0.617006	0.143469	4.300618
τ	0.097431	0.080557	0.014310	5.629554
F	0.018750	0.020767	0.001251	16.594955

$RMSE_x = 0.02033$, $RMSE_n = 0.01703$, $\sigma_p/\sigma_a = 0.60$

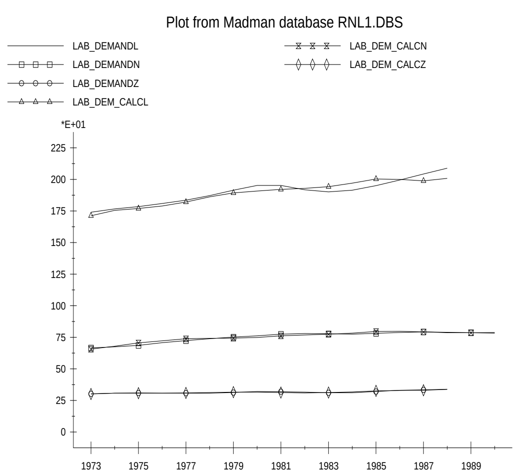


Fig. 12 Employment in the L, N and Z sectors

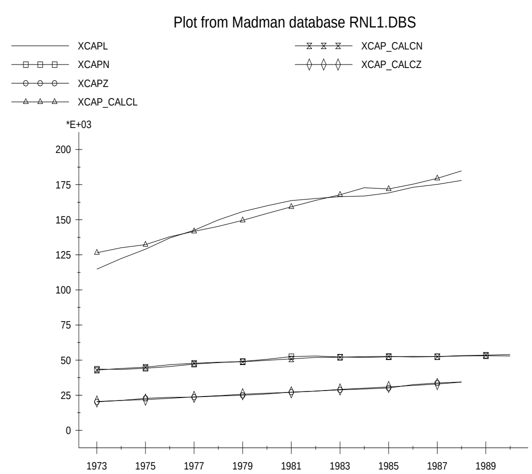


Fig. 13 Production capacity in the L, N and Z sectors

From the figures above, we can conclude that, as was the case with Germany, the L-sector shows a relatively high degree of serial correlation of the estimation residuals. This is especially apparent in case of production capacity. It should be noted here again that the rate of technical decay is very low for the L-sector : hence, the average characteristics of the capital stock have an inherent tendency to change relatively slowly over time.

4.3 Some Direct Comparisons between Germany and the Netherlands

In figures 14-19 below, we present some graphical information with regard to the technical change parameters for Germany and the Netherlands, as well as for the elasticities of substitution ex-ante and ex-post.

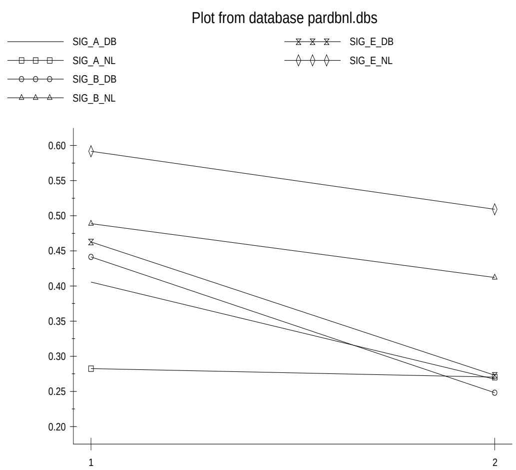


Fig. 14 Elasticities of Substitution Ex-Ante and Ex-Post in the A, B and E sectors

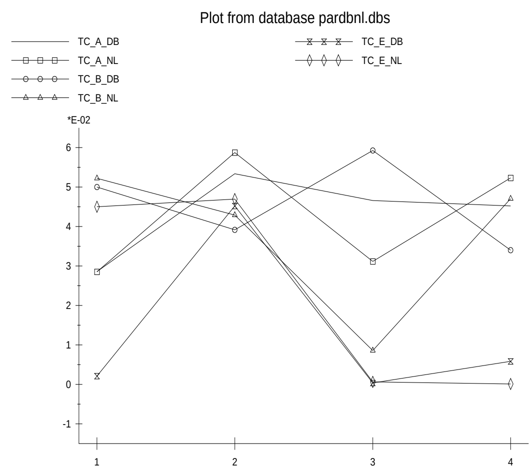


Fig. 15 Technical Change Parameters in the A, B and E sectors

In figure 14 we have depicted the values of the elasticities of substitution ex-ante and ex-post. The ex-ante values are depicted on the left (labelled 1), while the corresponding values ex-post are given on the right hand side of the graph (labelled 2). Germany has the post-fix 'DB', while the Netherlands have the post-fix 'NL'. The graphs associated with technical change represent $\gamma_I , \gamma_n , \mu_I , \mu_n$, which are labelled 1-4, respectively.

From figures 14, 16 and 18 we conclude first that the range of variation in the ex-ante elasticity of substitution in the services sectors is far smaller than in the case of the industrial sectors. This seems to point towards a greater similarity of the intrinsic characteristics of the production processes in the services sectors than in the industrial sectors. We notice too that, apart from the B-sector and the E-sector in case of the Netherlands, the elasticities of substitution ex-post all cluster around a value of about 0.3, whereas the average value of the elasticity of substitution ex-ante is equal to about 0.45. Hence, the overall elasticity of substitution (i.e. measured over both old and new capital) is of the order of 0.3, both in Germany and in the Netherlands.

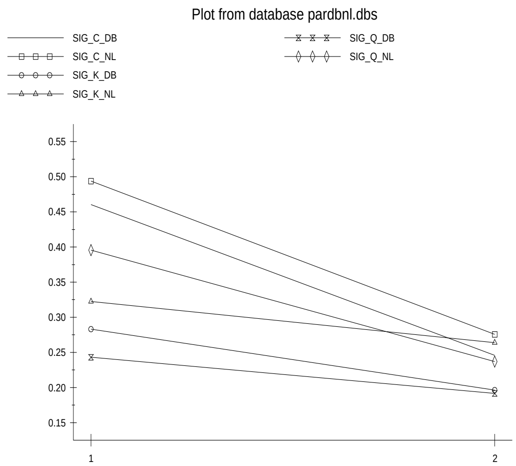


Fig. 16 Elasticities of Substitution Ex-Ante and Ex-Post in the C, K and Q sectors

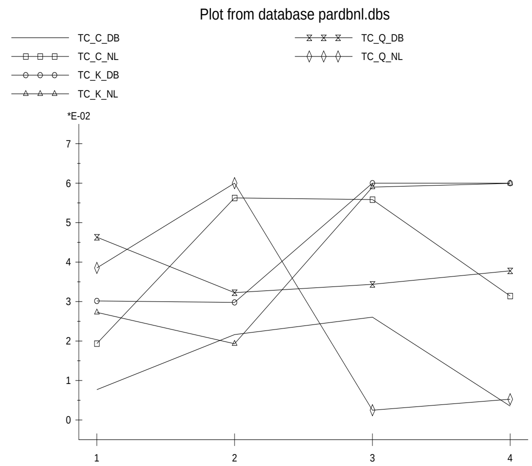


Fig. 17 Technical Change Parameters in the C, K and Q sector

With regard to the patterns and the levels of technical change, we may observe that on average there does not seem to be a structural tendency for technical change to be either mainly embodied or mainly disembodied : there is no relative peak at either labels 1 and 2, or labels 3 and 4. Secondly, with some exceptions, the biases in technical change have the same sign in Germany as well as in the Netherlands, although there can be differences in average levels. Examples of this kind are the C-sector and the K-sector in figure 17, as well as the L- and Z-sector in figure 19, while the N-sector provides a counter example in figure 19. Figure 15 shows a somewhat more mixed picture in this respect.

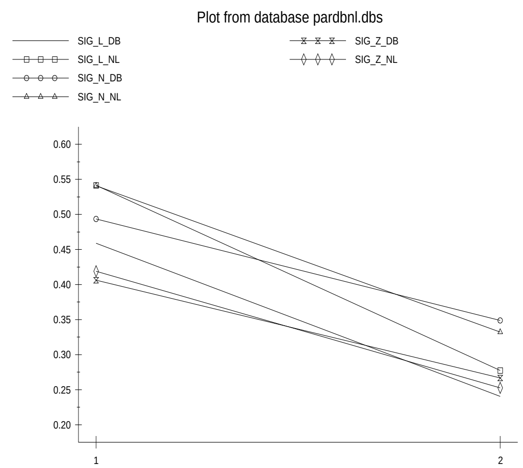


Fig. 18 Elasticities of Substitution Ex-Ante and Ex-Post in the L, N and Z sectors

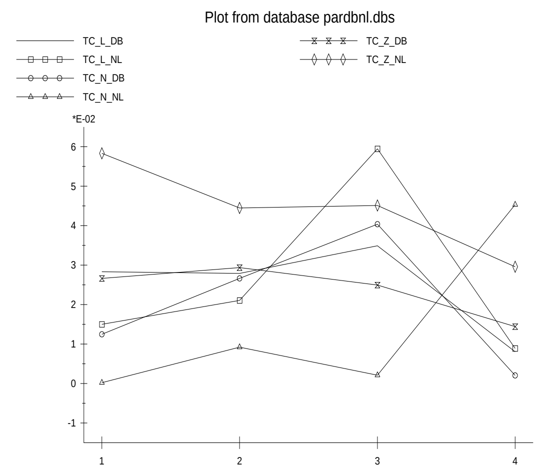


Fig. 19 Technical Change Parameters in the L, N and Z sector

4.4 Concluding Remarks

Looking over the figures presented in the previous section, one can conclude that the E-sector in case of the Netherlands is the only serious anomaly in terms of its fit. For the other sectors, both in Germany and the Netherlands, the fit is much better. This is especially apparent in case of the B-sector, which shows relatively large fluctuations both in terms of employment and in terms of production capacity, which are nonetheless captured quite well by the model. This also goes for the C-sector and the K-sector, and to a lesser extent for the Q-sector. Since these sectors experience relatively high values of the rate of technical decay, this might point to the practical importance of lagged adjustments (as these would be caused by a long 'effective' vintage history as implied by a low value of the rate of technical decay) for the ability of the RUM-model to duplicate real-life short term fluctuations. The latter is highlighted by the performance of the RUM model in the case of the services sectors, where serial correlation of the errors seems to be a problem which might in part be explained by the relatively low values of the rate of technical decay for the L- and the N-sector.

We can also (tentatively) conclude from the estimation results presented above that, in practice, the rates of embodied and disembodied technical change vary more between sectors than between countries. In addition, there seems to be no clear cut bias in favour of either embodied technical change or disembodied technical change. In practice this implies that the impact of embodied technical change at the aggregate level is on average less outspoken than the impact of disembodied technical change, since the latter type of technical change affects a much larger part of the aggregate factors of production than embodied technical change (which affects only the factors at the margin). Hence, our results seem to point out that on average the embodiment of technical change may not matter that much in practice. However, this is all but true, since the embodiment of technical change in the strict sense of the word is but one factor in the explanation of substitution possibilities and hence factor proportions ex-post. The results we have obtained indicate that the 'broad' technological characteristics of investment as a source of 'localised' and limited substitution possibilities ex-post, may be a far more important determinant of the average characteristics of the production process than either form of technical change.

5 Summary and Conclusion

In this paper we have set out an intertemporal putty-semi-putty vintage model called 'RUM'. RUM combines limited but non-zero substitution possibilities ex-post with a set of recursive adjustment / update rules regarding average capital productivity and the average value of the labour / capital ratio. These rules describe the changes in the average technological characteristics of the capital stock in terms of the changes in the technological characteristics of both existing capital and new capital goods. In this way we are able to integrate the notions of embodied and disembodied technical change into a quasi-vintage model which is a very good approximation of a full putty-semi-putty vintage model, although it requires only a fraction of the computational overhead which is usually associated with a full (putty-semi-putty) vintage model.

In RUM, producers do not only choose a technique on some pre-existing iso-quant, but since they know that their substitution possibilities ex-post are smaller than the ones ex-ante, they also select entire ex-post iso-quants by choosing the appropriate initial technique in the ex-post iso-quant field, conditional on the requirement that all new ex-post iso-quants at some moment of time are 'enveloped' by the ex-ante iso-quant. The latter iso-quant shifts through the labour-capital space due to embodied technical change. Ex-post iso-quants, once they have been chosen, shift through that same space too, but this is due to disembodied technical change.

In this paper we have shown that an ex-post iso-quant is uniquely defined by the point of tangency of the ex-post iso-quant in question and the 'enveloping' ex-ante iso-quant. Hence, choosing an optimum production technique now comes down, first, to choosing an optimum 'tangential technique', and secondly an optimum entry-technique on the ex-post iso-quant which is uniquely defined by that 'tangential technique'. These choices are made in an intertemporal setting where we show that the allocation of labour among both new and old vintages of capital are ruled by a putty-semi-putty equivalent of the Malcomson scrapping condition.

Using the RUM model as a representation of a full putty-semi-putty vintage model with linear homogeneous CES production functions both ex-ante and ex-post, we showed that it is not strictly necessary to engage in extensive vintage 'book-keeping' exercises. Rather, the application of a very limited set of recursive update rules with respect to the aggregate capital productivity and the aggregate labour / capital ratio in function of the characteristics of the new vintage to be installed, is sufficient to reproduce virtually all the relevant information generated by the full putty-semi-putty vintage model as has been illustrated in van Zon (1994).

We have estimated the parameters of the RUM-model for 9 different sectors of industry for Germany and the Netherlands. The results obtained indicate first that variations in the rate of technical change are larger among sectors than among countries, secondly that the estimated values of the rates of embodied and disembodied technical change are on average about equal in size, and third that the average value of the elasticity of substitution ex-post is about one third less than the average value of the elasticity of substitution ex-ante. Hence, we tentatively conclude that in practice and on average the real importance of the distinction between different vintages lies in the 'localisation' of substitution possibilities ex-post due to the choice of a specific 'tangential technique', although in individual cases the distinction between embodied and disembodied technical change may still be an important factor in the determination of the average characteristics of the production process.

An important theoretical feature of the RUM-model is that it can actually distinguish between the different determinants of the characteristics of the production process. But it's most important feature is that RUM allows us to handle these determinants in such a way that it can serve as a comprehensive and manageable alternative to the large computational and 'book-keeping' burden which a standard vintage modelling approach usually entails.

Appendix A : Some Algebra

From equation (9.D) we have :

$$p'_i \cdot \frac{(1-\delta)^{i-j}}{\kappa_{j,i}} \cdot \left(1 - \frac{w_i}{p_i} \cdot \nu_{j,i}\right) = \left(\lambda^n_{j,i} \cdot \frac{\partial f^{j,i}}{\partial \kappa_{j,i}} + \lambda^x_i \cdot \frac{Y_{j,i}}{\kappa_{j,i}}\right) \cdot \frac{(1-\delta)^{i-j}}{Y_{j,i}} \tag{A.1}$$

Substitution of (A.1) into (9.E) leads directly to :

$$q'_j = \sum_{i=j}^{\infty} \lambda^n_{j,i} \cdot \frac{\partial f^{j,i}}{\partial \kappa_{j,i}} \cdot \frac{(1-\delta)^{i-j}}{Y_{j,i}} = \sum_{i=j}^{\infty} (p'_i - \lambda^x_i) \cdot \frac{\partial f^{j,i}}{\partial \kappa_{j,i}} \cdot (1-\delta)^{i-j} \tag{A.2}$$

by means of straightforward substitution of (13) into the first part of (15).

Equation (23) says :

$$q'_j = \sum_{i=j}^{\infty} \lambda^n_{j,i} \cdot \frac{\partial f^{j,i}}{\partial \kappa_{j,i}} \cdot \frac{(1-\delta)^{i-j}}{Y_{j,i}} = \sum_{i=j}^{\infty} (p'_i - \lambda^x_i) \cdot \frac{\partial f^{j,i}}{\partial \kappa_{j,i}} \cdot (1-\delta)^{i-j} \tag{A.3}$$

Hence :

$$q'_{j+1} = \sum_{i=j+1}^{\infty} (p'_i - \lambda^x_i) \cdot \frac{\partial f^{j,i}}{\partial \kappa_{j,i}} \cdot (1-\delta)^{i-j-1} \tag{A.4}$$

Since $q'_{j+1} = (1+\hat{q}_j) \cdot (1+r_t)^{-1} \cdot q'_j$ (where $\hat{q}$ is the rate of growth of q), it follows immediately that :

$$q'_j - (1-\delta) \cdot q'_{j+1} = q'_j \cdot \left(1 - \frac{(1-\delta) \cdot (1+\hat{q}_j)}{1+r_t}\right) = (p'_j - \lambda^x_j) \cdot \frac{\partial f^{j,j}}{\partial \kappa_{j,j}} \tag{A.5}$$

Multiplication of (A.5) by $\kappa_{j,j}$ and using the Euler equation, gives :

$$q'_j \cdot \kappa_{j,j} \cdot \left(1 - \frac{(1-\delta) \cdot (1+\hat{q}_j)}{1+r_t}\right) = (p'_j - \lambda^x_j) \cdot \left(1 - \frac{\partial f^{j,j}}{\partial \nu_{j,j}} \cdot \nu_{j,j}\right) = p'_j - \lambda^x_j - w'_j \cdot \nu_{j,j} \quad \Rightarrow \tag{A.6}$$

$$p'_j - \lambda^x_j = q'_j \cdot \kappa_{j,j} \cdot \left(1 - \frac{(1-\delta) \cdot (1+\hat{q}_j)}{1+r_t}\right) + w'_j \cdot \nu_{j,j}$$

where we have substituted equations (9.B) and (21) into the expression for the marginal productivity of labour in the second part of (A.6). Note that (A.6) reproduces the familiar expression for the user cost of capital per period as the sum of the rate of technical decay and the rate of interest less the proportionate capital gains on existing machinery. Note too, that $p'_i - \lambda_i^x$ measures the user cost of both labour and capital per unit of output on the newest equipment.

Appendix B. The Sectoral Classification

ID	Sector	NACE R44	NACE R25
A	Agriculture	01	01
B	Building and Construction	53	53
C	Consumer Goods	31,33,35,37,39,41,43,45,47,49,51	36,42,47,48,49
E	Energy	03,05,07,09,11	06
K	Capital Goods	19,21,23,25,27,29	19,21,23,25,28
L	Commercial Services	55,57,59,69A,71,73,75,77,79	56,59,69A,74
N	Non-Market Services	81,85,89,93	86
Q	Intermediate Goods	13,15,17	13,15,17
Z	Communication and Transportation	61,63,65,67	61,63,65,67

The description of the NACE classifications can be found in ESA (1979).

References

- d'Alcantara, G. And A. Italianer, 1982, European Project for a multinational macrosectoral model, report EUR 9443, Commission of the European Communities, Directorate General for Science, Research and Development.
- Bischoff, C.W., 1971, 'The effect of alternative lag distributions', in G. Fromm (ed.), Tax incentives and capital spending, the Brookings Institution, Washington.
- Bunday, B.D. and G.R. Garside, 1987, Optimisation Methods in Pascal, Edward Arnold, London, pp. 98-107.
- Eigenraam, C.J.J., 1987, 'Een produktiefunctie voor heterogene jaargangen en daarbij behorende factorvraagvergelijkingen', Onderzoeksmemorandum Centraal Planbureau, No. 34, Den Haag.
- ESA, 1979, European System of Integrated Economic Accounts, Second Edition, Eurostat, Luxemburg.
- Gelauff, G.M.M., Wennekers, A.R.M. and A.H.M. de Jong, 1985, 'A putty-clay model with three factors of production and partly endogenous technical progress', De Economist, Vol. 133, pp. 327-351.
- Goldberg, D.E., 1989, Genetic Algorithms in Search, Optimization, and Machine Learning, Addison-Wesley, Massachusets.
- Hartog, H. den, and H.S. Tjan, 1976, 'Investment, Wages, Prices and Demand for Labour', de Economist, 124, pp. 32-55.
- IMF, 1991, International Financial Statistics Yearbook, Washington DC.
- Johansen, L., 1959, 'Substitution versus Fixed Production Coefficients in the Theory of Economic Growth', Econometrica, vol. 27, pp. 157-175.
- Kuipers, S.K. and A.H. van Zon, 1982, 'Output and Employment Growth in the Netherlands in the Postwar Period: A Putty-Clay Approach', de Economist, 130, pp. 38-70.
- Malcomson, J.M., 1975, 'Replacement and the rental value of capital equipment subject to obsolescence', Journal of Economic Theory, 10, pp. 24-41.
- Meijers, H.H.M. and A.H. van Zon, 1991, 'A Reformulation of the HERMES Vintage Production Structure', MERIT Research Memorandum, no. 91-016.
- Meijers, H.H.M., 1994, 'On the Slowdown of Productivity Growth in the Netherlands 1960-1988: a Macro-Economic Vintage Diffusion Approach', Economics of Innovation and New Technology, forthcoming.

- Muysken, J. and A.H. van Zon, 1987, 'Employment and unemployment in the Netherlands, 1960-1984 : a putty-clay approach', Recherches Economiques de Louvain, Vol. 53, pp. 101-133.

- Salter, W.E.G., 1960, Productivity and Technical Change, Cambridge University Press.

- Zon, A.H. van, 1986, A Simple Multi-Sector Model of the Dutch Economy for the Period 1950-1968, Doctoral Thesis, Groningen.

- Zon, A.H. van, 1993, 'The Continuous Recursive Adjustment Putty-Putty Model: An Outline of Its Main Features', MERIT Research Memorandum, 93-016.

- Zon, A.H. van, 1994, 'RUM: a Simple Recursive Update Model Providing a Condensed Representation of a Putty-Semi-Putty Vintage Model', MERIT Research Memorandum, 94-002.

Table of Contents